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A COMPARATIVE STUDY ON REALIZATION OF CHUA'S CIRCUIT: HYBRID REALIZATIONS OF CHUA'S CIRCUIT COMBINING THE CIRCUIT TOPOLOGIES PROPOSED FOR CHUA'S DIODE AND INDUCTOR ELEMENTS

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Several realizations of Chua's circuit have been proposed in the literature. The methodologies used in these realizations can be divided into two basic categories. In the first approach, a variety of circuit topologies have been considered for realizing the nonlinear resistor, N_R in Chua's circuit. The main idea in the second approach related to the implementation of the circuit is an inductorless realization of Chua's circuit. For this purpose, several circuit topologies serving as a synthetic inductor, i.e. inductance simulator, are used instead of the inductor element in Chua's circuit. In this study, after comparing the circuit topologies proposed for Chua's diode and the inductor element in the circuit, we will present six alternative hybrid realizations of Chua's circuit combining circuit topologies proposed for the nonlinear resistor and the inductor element in the literature. The PSpice simulation experiments confirm that the six alternative hybrid realizations of the circuit exhibit the original chaotic behavior of Chua's circuit.

Keywords: Chua's circuit; Chua's diode; inductance simulators; current feedback op amp; four terminal floating nullor; Wien bridge oscillator.

1. Introduction

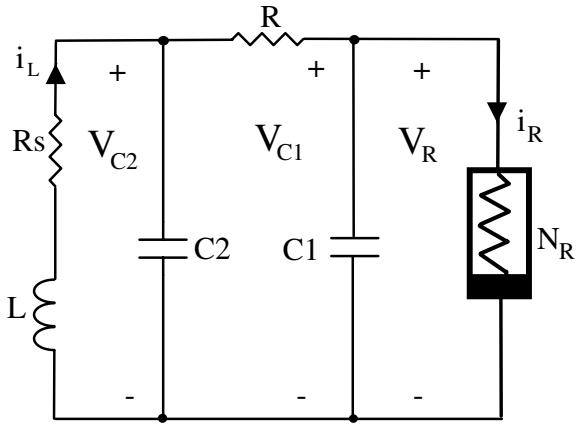
Until now, there has been considerable interest in the design and implementation of chaotic circuits and many such circuits have been proposed, see for example [Abuel'matti & Aiyad, 1995; Elwakil & Kennedy, 2001; Elwakil & Soliman, 1997; Kiliç *et al.*, 2000; Matsumoto, 1984; Murali *et al.*, 1994; Nakagawa & Saito, 1996; Namajunas & Tamasevicius, 1996; Varrientos & Sanchez-Sinencio, 1998] and references therein. Among them, Chua's chaotic circuit [Chua *et al.*, 1993] has a special position. Since the circuit is an extremely simple system, and yet it exhibits a rich variety of bifurcations and chaos, it has been studied extensively and accepted as a paradigm for studying with chaos and chaotic signals.

Chua's circuit, shown in Fig. 1(a), contains three energy storage elements, a linear resistor and a nonlinear resistor N_R, namely Chua's diode. Chua's circuit is defined by the following state equations:

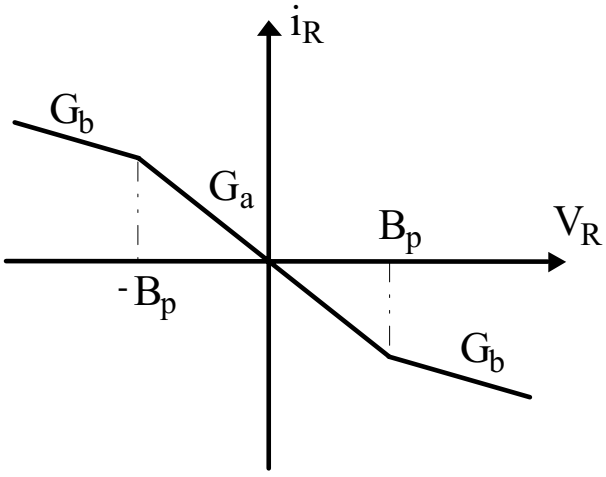
$$L\frac{di_L}{dt} = -V_{C2} - i_L \cdot R_S \tag{1}$$

$$C_2\frac{dV_{C2}}{dt} = i_L - \frac{1}{R}(V_{C2} - V_R) \tag{2}$$

$$C_1\frac{dV_R}{dt} = \frac{1}{R}(V_{C2} - V_R) - f(V_R) \tag{3}$$



(a)



(b)

Fig. 1. (a) Chua's circuit. (b) The three-segment piecewise linear characteristic of the nonlinear resistor in Chua's circuit.

where $f(V_R)$ is a piecewise-linear function, shown graphically in Fig. 1(b), defined by

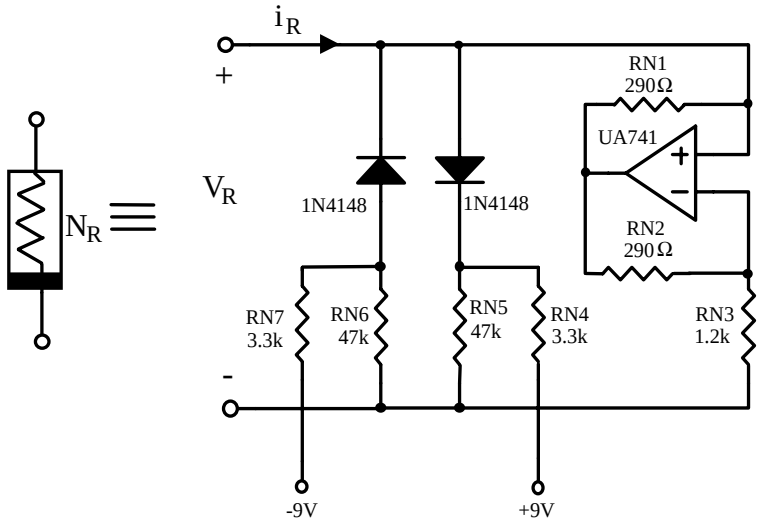
$$i_R = f(V_R) = G_b V_R + \frac{1}{2} \cdot (G_a - G_b) \times (|V_R + B_p| - |V_R - B_p|) \quad (4)$$

where G_a and G_b are the slopes in the inner and outer regions, respectively, and $\pm B_p$ denote the breakpoints.

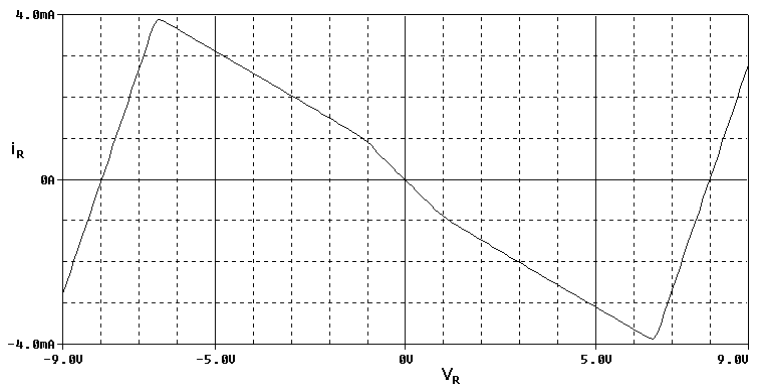
Several realizations of Chua's circuit have been proposed in the literature. The methodologies used in these realizations can be divided into two basic categories. In the first approach, a variety of circuit topologies have been considered for realizing the nonlinear resistor, N_R in Chua's circuit. In early implementations, Chua's diode has been realized using circuit elements such as diodes [Matsumoto, 1985], Op Amps [Zhong & Ayrom, 1985; Kennedy, 1992], transistors [Matsumoto *et al.*, 1986] and OTAs [Cruz & Chua, 1992]. Of these approaches, Kennedy's op amp-based nonlinear resistor [Kennedy, 1992], which is formed by connecting two voltage-controlled negative impedance converters in parallel, has been accepted as a standard. This realization uses two conventional voltage op amps (VOA) and six resistors. Recently, implementations of Chua's diode using the current feedback op amps (CFOA) have been developed by Senani and Gupta [1998] and Elwakil and Kennedy [2000]. In both of these implementations, the attractive combined voltage–current capabilities of a CFOA are used to synthesize Chua's diode. Although these two implementations are slightly different, both of them require two fewer resistors than Kennedy's [1992] implementation. A comparative investigation of the implementations of Chua's diode in the literature will be given in Sec. 2.

The main idea in the second approach related to the implementation of Chua's circuit is inductorless realization of the circuit. For this purpose, various circuit topologies providing a synthetic inductor, i.e. by inductance simulation, are used instead of the inductor in Chua's circuit. The inductance simulators used in such realizations are generally based on circuit elements such as op amps [Torres & Aguirre, 2000] and OTAs [Cruz & Chua, 1993; Rodriguez-Vazquez & Delgado-Restituto, 1993]. Recently, a CFOA-based synthetic inductor [Senani & Gupta, 1998] and an FTFN-based inductance simulator [Kiliç *et al.*, 2002] have been proposed for an inductorless Chua's circuit and other chaotic circuits. In addition to using an inductance simulator for an inductorless Chua's circuit, another approach has been developed based on replacing the LC resonator of Chua's circuit with an RC configuration. Such a realization was presented by Morgül [2000]. In Morgül's implementation, the LC resonator of Chua's circuit was replaced by a Wien bridge-based RC configuration. In Sec. 3, this second category of approaches using an inductorless Chua's circuit will be investigated.

After presenting the comparative investigation of the circuit topologies proposed for Chua's diode and the inductor in the circuit, in Sec. 4 we will present six alternative realizations of Chua's circuit combining circuit topologies proposed in the



(a)



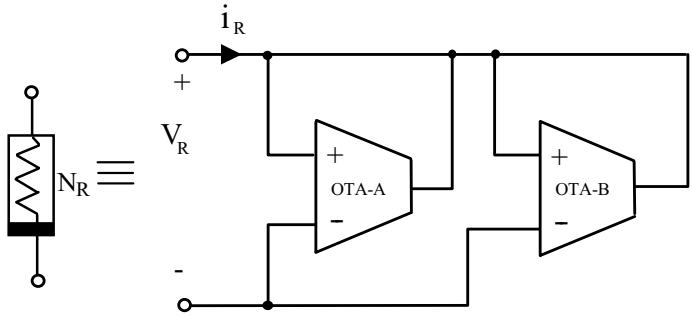
(b)

Fig. 2. (a) The circuit structure of Chua's diode proposed by Matsumoto *et al.* [1985]. (b) Simulated v–i characteristic of Chua's diode of Fig. 2(a).

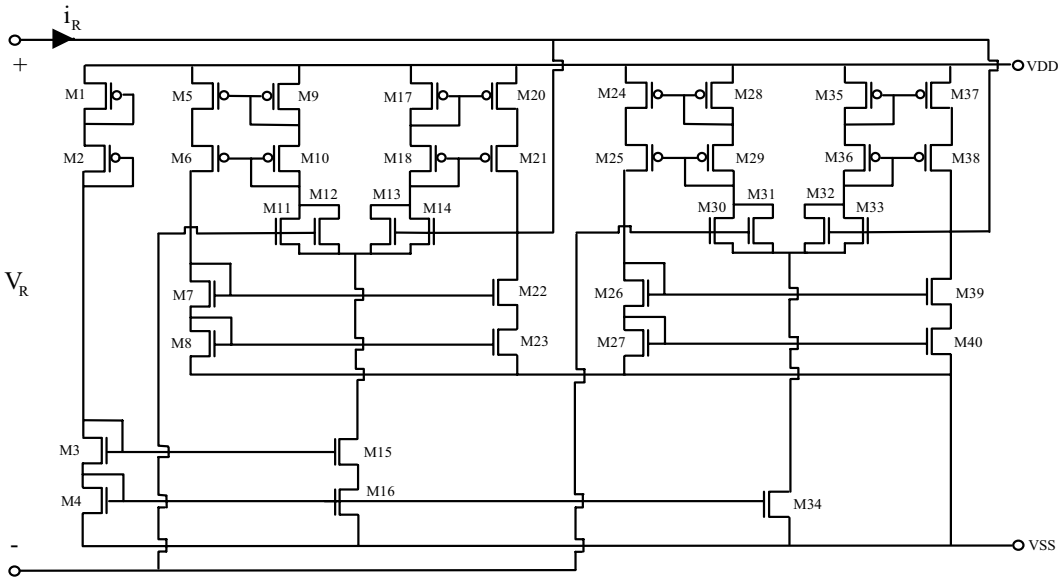
literature for the nonlinear resistor and inductor elements. As the circuit topologies proposed for the nonlinear resistor and inductor in Chua's circuit are used together in our alternative realizations, we call these "**Hybrid Realizations**". The PSpice simulation experiments confirm that the six alternative hybrid realizations of Chua's circuit exhibit the original chaotic behavior of Chua's circuit. We hope that these hybrid realizations will provide alternative solutions for readers studying Chua's circuit. In Sec. 5, we will give some concluding remarks.

2. Circuit Topologies for Realization of Chua's Diode

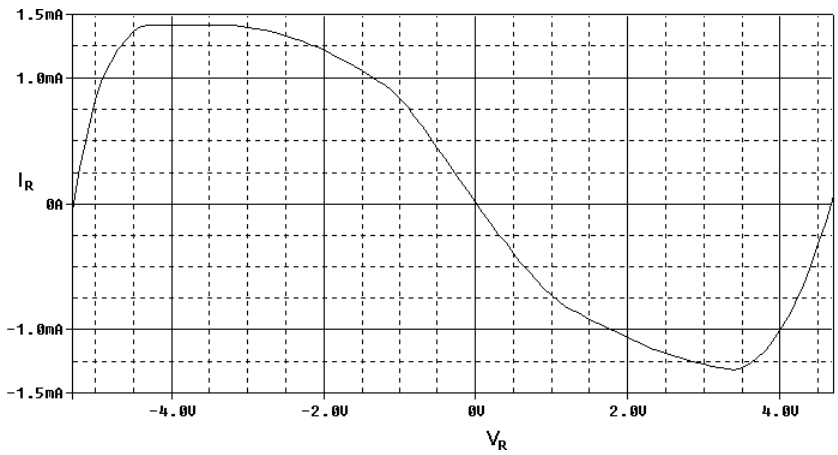
Several implementations of Chua's diode already exist in the literature. Early implementations use diodes [Matsumoto *et al.*, 1985], Op Amps [Zhong & Ayrom, 1985; Kennedy, 1992], transistors [Matsumoto *et al.*, 1986] and OTAs [Cruz & Chua, 1992]. One of the earliest implementations of Chua's diode implemented by Matsumoto *et al.* [1985] is shown in Fig. 2(a). As shown in the figure, Chua's diode is realized by means of an op amp with a pair of diodes, seven resistors and DC supply voltages of $\pm 9\,\mathrm{V}$, yielding $G_a \cong -0.8$ mA/V, $G_b \cong -0.5$ mA/V and $\pm B_p = \pm 1\,\mathrm{V}$. The simulated v–i characteristic of Chua's diode of Fig. 2(a) is shown in Fig. 2(b). Cruz and Chua [1992] designed the first monolithic implementation of Chua's diode using the OTA-based circuit topology. The OTA-based structure and transistor schematic of the nonlinear resistor are shown in Figs. 3(a) and 3(b), respectively. As indicated



(a)



(b)

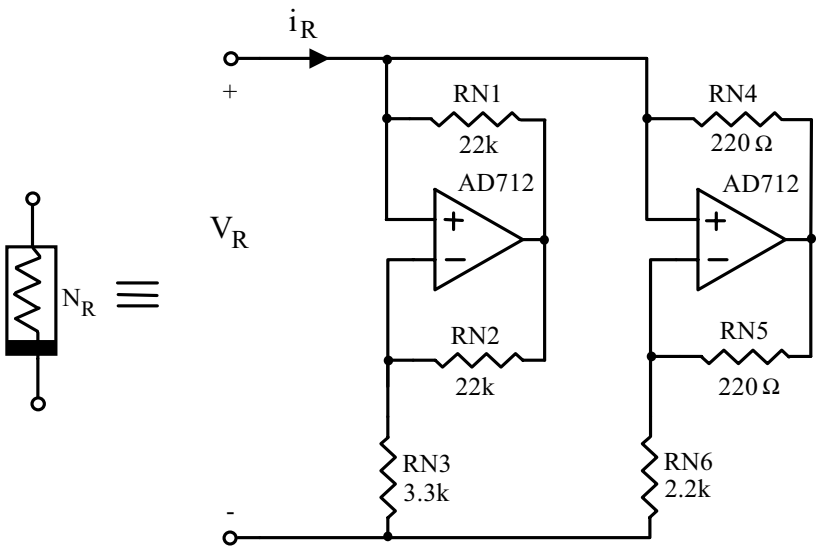


(c)

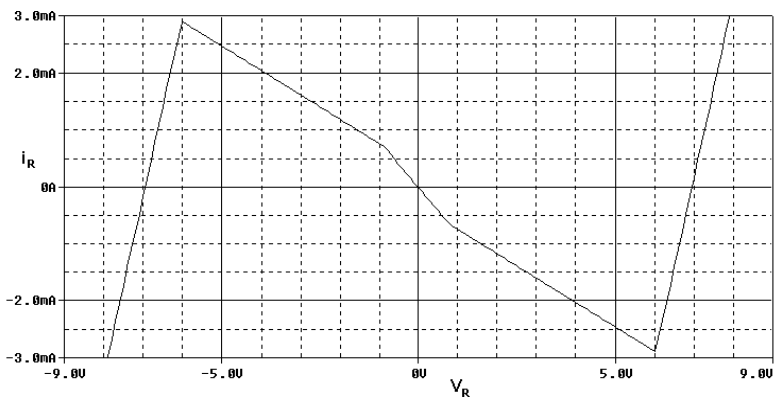
Fig. 3. (a) The OTA-based nonlinear resistor structure implemented by Cruz and Chua [1992]. (b) Transistor schematic of the OTA-based nonlinear resistor in Fig. 3(a). (c) Simulated v–i characteristic of the nonlinear resistor of Fig. 3(b).

in Fig. 3(a), this realization is based on only two OTAs and the dc parameters are determined as $G_a \cong -0.78$ mA/V, $G_b \cong -0.41$ mA/V and $\pm Bp = \pm 0.7$ V. Using the technological data and parameters given in [Cruz & Chua, 1992], the simulated v–i characteristic of Chua's diode of Fig. 3(b) is shown in Fig. 3(c). The design process of two suitable transconductance amplifiers can be found in detail in [Cruz & Chua, 1992]. A similar monolithic implementation of Chua's diode was realized by Rodriguez-Vazquez *et al.* [1993]. A realization of Chua's diode, proposed by Kennedy [1992], which is designed by connecting two voltage-controlled negative impedance converters in parallel, has been accepted as the standard for discrete implementation. The op amp-based nonlinear resistor structure with its simulated dc characteristic is shown in Fig. 4. As shown in Fig. 4(a), Kennedy's realization uses two op amps, operating in both their linear and nonlinear regions, and six resistors. The slopes and breakpoints are chosen as $G_a \cong -0.756$ mA/V, $G_b \cong -0.409$ mA/V and $\pm B_p = \pm 1$ V with the circuit parameters in Fig. 4(a). Since it has a simple and easily configurable circuit structure, most of the experimental studies with Chua's circuit in the literature have been performed using Kennedy's implementation.

Due to the frequency limitations of the voltage op amps, Kennedy's implementation imposes an upper limit on the operating frequency. Therefore, in the literature new design ideas for implementing Chua's diode are considered aiming for high

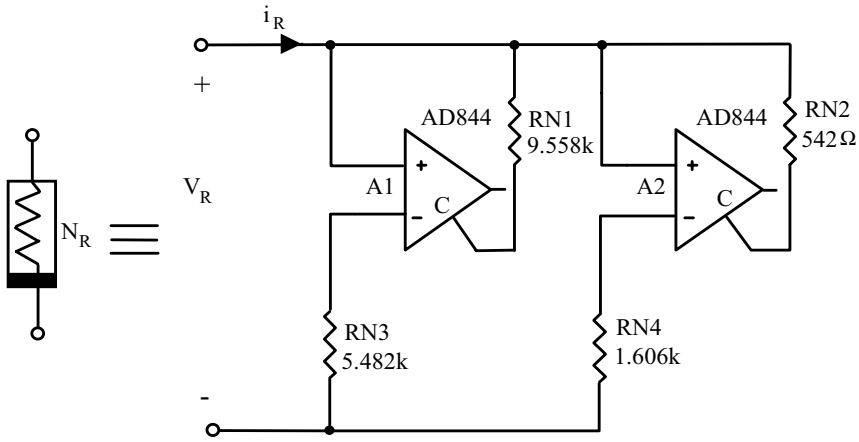


(a)

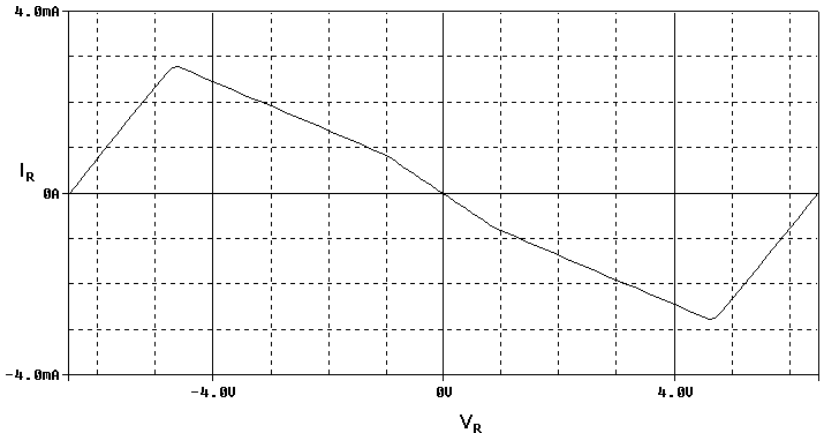


(b)

Fig. 4. (a) The op amp-based nonlinear resistor structure proposed by Kennedy [1992]. (b) Simulated v–i characteristic of the nonlinear resistor of Fig. 4(a).



(a)

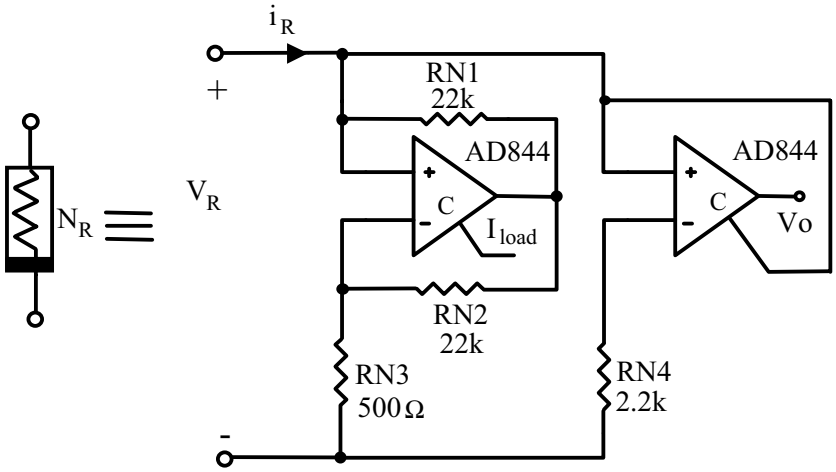


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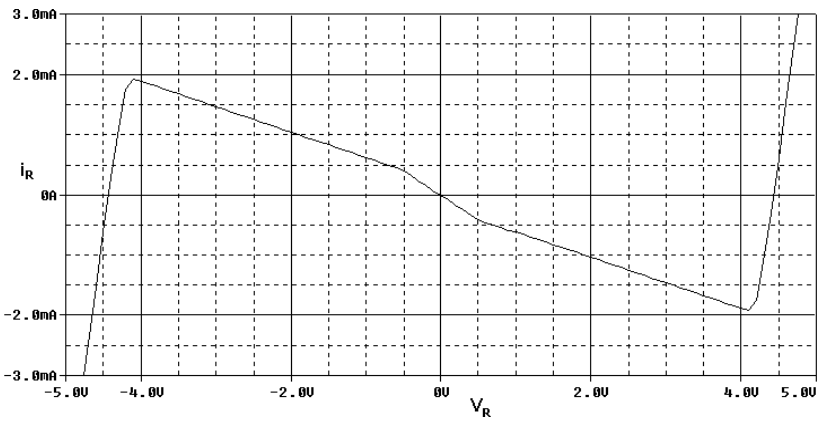
Fig. 5. (a) The CFOA-based nonlinear resistor structure proposed by Senani and Gupta [1998]. (b) Simulated v–i characteristic of the nonlinear resistor of Fig. 5(a).

frequency chaotic signals. Recently, two alternative implementations of Kennedy's circuit have been presented by Senani and Gupta [1998] and Elwakil and Kennedy [2000]. The proposed nonlinear resistor circuit topologies are shown in Figs. 5 and 6, respectively. In these implementations, the authors aim to use the voltage–current capabilities of a current feedback op amp (CFOA) which offer several advantages over a classic voltage op amp (VOA). In the circuit topology shown in Fig. 5(a), presented by Senani and Gupta [1998], each of the CFOAs is configured as a negative impedance converter, with resistors R_{N1} and R_{N2} shorted. In this case the circuit is basically a parallel connection of two negative resistors $-R_{N3}$ and $-R_{N4}$. Adding resistors R_{N1} and R_{N2} and using different power supply voltages for the two CFOAs, the authors offer the circuit realization for Chua's diode with the following parameters: $R_{N1} = 9.558$ kΩ, $R_{N2} = 542$ kΩ, $R_{N3} = 5.482$ kΩ, $R_{N4} = 1.606$ kΩ; and the power supplies $\pm V_1 = \pm 4.05$ V, $\pm V_2 = \pm 11.23$ V for A1 and A2, respectively. The use of these parameters yields $G_a \cong -0.8$ mA/V, $G_b \cong -0.5$ mA/V and $\pm B_p = \pm 1$ V. The simulated v–i characteristic of Chua's diode of Fig. 5(a) is shown in Fig. 5(b). The authors have experimentally verified this realization using AD844 type CFOAs. Senani & Gupta [1998] also designed a CFOA-based synthetic inductor. We will discuss the circuit topology of the synthetic inductor in Sec. 3.

In Elwakil and Kennedy's [2000] CFOA-based implementation of Chua's diode, shown in Fig. 6 with its simulated dc characteristic, the design methodology is similar to Kennedy's circuit's



(a)



(b)

Fig. 6. (a) The CFOA-based nonlinear resistor structure proposed by Elwakil and Kennedy [2000]. (b) Simulated v–i characteristic of the nonlinear resistor of Fig. 6(a).

[1992]. While the authors used a CFOA connected as a CCII with resistor R_{N4} to operate as a linear negative impedance converter (NIC) operating primarily in its linear region in Kennedy's circuit, they configured resistors R_{N1} , R_{N2}, R_{N3} and the associated CFOA to operate as a nonlinear voltage-controlled negative impedance converter (VNIC) instead of the VOA and its three resistors in Kennedy's implementation. While the authors used the same values of R_{N1}, R_{N2} and R_{N4} as in Kennedy's circuit, they determined R_{N3} by adjusting its value to observe chaos. Two AD844 type CFOAs biased with $\pm 9\,\mathrm{V}$ are used in this implementation. With these parameters, the desired dc characteristics, $G_a \cong -0.8$ mA/V and $G_b \cong -0.5$ mA/V , have been achieved.

Both Senani and Gupta's design and Elwakil and Kennedy's design for Chua's diode employ fewer resistors than the six resistors in Kennedy's original design. Elwakil and Kennedy's design provides a buffered output voltage that directly represents a state variable. Since one of the two outputs voltages can be used as the carrier signal in chaotic communication systems, the feature of a buffered and isolated voltage output directly representing a state variable in the Chua's diode design of Elwakil and Kennedy constitutes an advantage over Senani and Gupta's design. Both Elwakil and Kennedy's and Senani and Gupta's circuit topologies can be configured such that chaotic spectrum is extended to higher frequencies than Kennedy's original implementation.

3. Circuit Topologies for Inductorless Chua's Circuit

An inductorless Chua's circuit can be produced by using a synthetic inductor, i.e. an inductance simulator, instead of inductor element, and by using RC configurations instead of the LC resonator in Chua's circuit. Inductorless realizations of Chua's circuit using OTA-based inductance simulators are reported in [Cruz & Chua, 1993; Rodriguez-Vazquez & Delgado-Restituto, 1993]. In addition to OTA-based realizations, op amp-based inductance simulator structures can be used in designing inductorless Chua's circuits and other chaotic circuits. Such a design was proposed by Torres and Aguirre [2000]. The proposed inductance simulator, shown in Fig. 7, is a gyrator circuit built as an impedance converter.[1] The equivalent inductance value can be computed as follows:

$$L_{\text{eq}} = \frac{R_1 R_3 R_4 C_3}{R_2} \tag{5}$$

The authors configured the inductance simulator circuit parameters as $R_1 = R_2 = R_3 = 1$ kΩ, $R_4 = 1.8$ kΩ, $C_3 = 23.5$ μF giving an inductance value of $L_{\text{eq}} = 42.3$ H. Large values of inductance L and low frequency chaos can be accomplished easily in this way. Due to the nonidealities of the op amps, this approach has a limited frequency range. Therefore, in experimental studies with such op amp-based inductance simulators, the nonidealities and parasitics should be considered for the inductor. An additional drawback is that this op amp-based inductor simulator can only be used for grounded inductance as in Chua's circuit. For simulating a floating inductance in any chaotic circuit, alternative topologies should be used.

Senani and Gupta [1998] designed a synthetic inductor based on a CFOA for an inductorless realization of Chua's circuit. The synthetic inductor designed by Senani and Gupta is shown in Fig. 8. In this inductance simulator, the equivalent inductance value can be computed as follows:

$$L_{\text{eq}} = C_3 R_1 R_2 \tag{6}$$

Considering the parasitic impedance of the CFOAs, the authors determined the circuit parameter values as $R_1 = 3.744$ kΩ, $R_2 = 1.459$ kΩ, and $C_3 = 5$ nF giving an inductance value of $L_{\text{eq}} = 28.53$ mH.

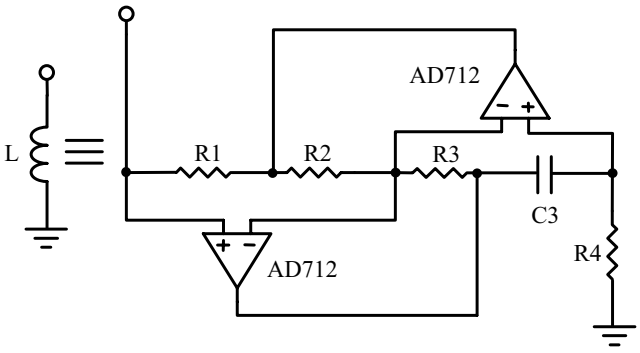


Fig. 7. Op amp-based inductance simulator.

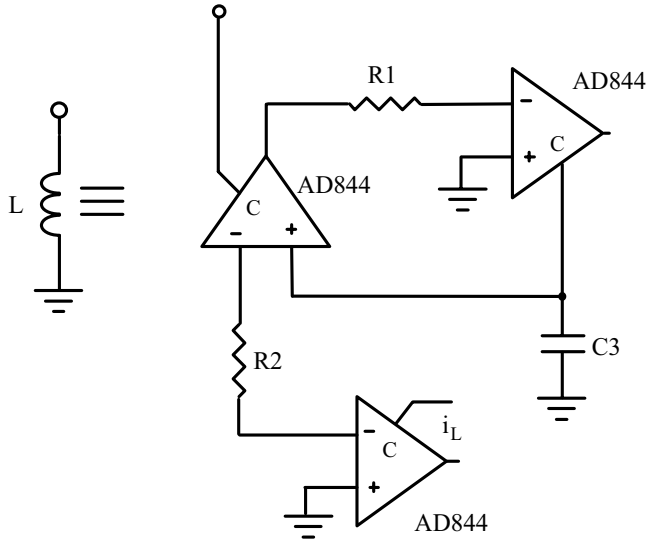


Fig. 8. CFOA-based synthetic inductor proposed by Senani and Gupta [1998].

Senani and Gupta [1998] have used the CFOA as an active circuit element for realizing both Chua's diode and a synthetic inductor. This element has a significant advantage for experimental studies. Although this inductance simulator employs three CFOAs, it uses the minimum number of passive components. While in Torres and Aguirre's [2000] op amp-based design for simulating the inductor in Chua's circuit, the third state variable (the inductor current i_L in Chua's circuit) cannot be determined in a direct manner, in Senani and Gupta's design, the inductor current i_L is available directly as a current output.[2]

[1] A gyrator-based inductor was implemented by Kennedy in Chua's laboratory in 1986.

[2] Kennedy's gyrator-based implementation [1986] also provides a voltage output which is proportional to the inductor current.

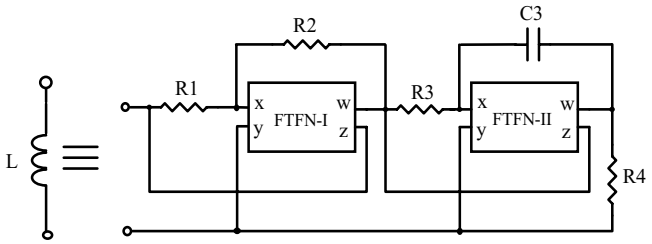


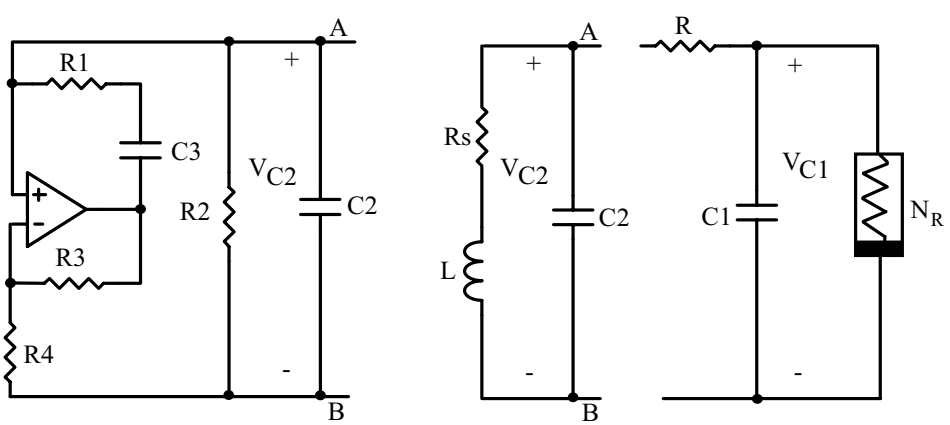
Fig. 9. FTFN-based inductance simulator.

Recently, we introduced an alternative inductance simulator topology based on a four terminal floating nullor (FTFN) for inductorless realization of Chua's circuit and other chaotic oscillators such as the mixed-mode chaotic circuit [Kiliç *et al.*, 2002] which contains both a grounded and a floating inductor element. The FTFN-based inductance simulator, shown in Fig. 9, allows one to simulate not only a grounded inductor but also a floating inductor. Although the FTFN-based inductance simulator structure is shown in floating inductor form in Fig. 9, this simulator may also be used as a grounded inductor by connecting one port of the floating inductance to ground. Routine analysis yields the equivalent inductance between the two terminals as:

$$L_{\text{eq}} = \frac{C_3 R_1 R_2 R_3}{R_4} \tag{7}$$

As there is a CMOS realization of the FTFN circuit topology [Çam *et al.*, 2000], the FTFN circuit can also be constructed using AD844s. Moreover, CMOS-based FTFN inductance simulation provides new possibilities to the integrated circuit designer for realizing not only Chua's circuit but also other chaotic circuits in monolithic form. The circuit element values of FTFN-based inductance simulator are determined as $R_1 = R_2 = R_3 = R_4 = 1\ \text{k}\Omega$ and $C_3 = 18$ nF to simulate an inductance of $L = 18$ mH in Chua's circuit.

In addition, to use inductance simulator for inductorless Chua's circuit, an alternative approach has been developed based on replacing the LC resonator of Chua's circuit with an RC configuration. Such a realization has been proposed by Morgül [2000]. There are other RC chaotic oscillators in the literature, see for example [Elwakil & Soliman, 1997; Namajunas & Tamasevicius, 1995, 1996] and references therein. In Morgül's implementation, shown in Fig. 10, the LC resonator of Chua's circuit was replaced by a Wien bridge-based circuit topology. Morgül [2000] showed that with appropriate element values, the Wien bridge-based circuit topology realizes Chua's circuit. Two modes (passive and oscillatory) have been identified in this implementation. In the passive mode, R, C_1 and the nonlinear resistor are kept the same in both circuits, and the input impedances of the Wien bridge part and the LC part in Fig. 10 are matched. In the oscillatory mode, the Wien bridge is first tuned to observe oscillations (i.e. when R is open-circuited), and then by tuning R it is possible to observe chaotic oscillations for certain parameter values. The element values of the Wien bridge-based circuit in Fig. 10 were determined as $R_1 = R_2 = R_4 = 100\ \Omega$, $R_3 = 200\ \Omega$, $C_2 = C_3 = 220$ nF and $C_1 = 2.2$ nF.



Wien bridge-based RC configuration LC-resonator of Chua's circuit

Fig. 10. Wien bridge-based circuit topology, proposed by Morgül [2000], for replacing with LC resonator of Chua's circuit.

Table 1. Hybrid realizations of Chua's circuit and the circuit topologies for the nonlinear resistor and inductor elements.

Hybrid Realizations of Chua's Circuit	The Circuit Topology Used in Hyrid Realization for Chua's Diode	The Circuit Topology Used in Hybrid Realization for the Inductor Element
Hybrid-I Realization	Kennedy's VOA-based nonlinear resistor [Kennedy, 1992]	Senani and Gupta's CFOA-based synthetic inductor [Senani & Gupta, 1998]
Hybrid-II Realization	Elwakil and Kennedy's CFOA-based nonlinear resistor [Elwakil & Kennedy, 2000]	Senani and Gupta's CFOA-based synthetic inductor [Senani & Gupta, 1998]
Hybrid-III Realization	Kennedy's VOA-based nonlinear resistor [Kennedy, 1992]	FTFN-based inductance simulator [Kiliç *et al.*, 2002]
Hybrid-IV Realization	Elwakil and Kennedy's CFOA-based nonlinear resistor [Elwakil & Kennedy, 2000]	FTFN-based inductance simulator [Kiliç *et al.*, 2002].
Hybrid-V Realization	Elwakil and Kennedy's CFOA-based nonlinear resistor [Elwakil & Kennedy, 2000]	VOA-based inductance simulator [Torres & Aguirre, 2000]
Hybrid-VI Realization	Elwakil and Kennedy's CFOA-based nonlinear resistor [Elwakil & Kennedy, 2000]	Morgül's Wien bridge-based RC configuration [Morgül, 2000]

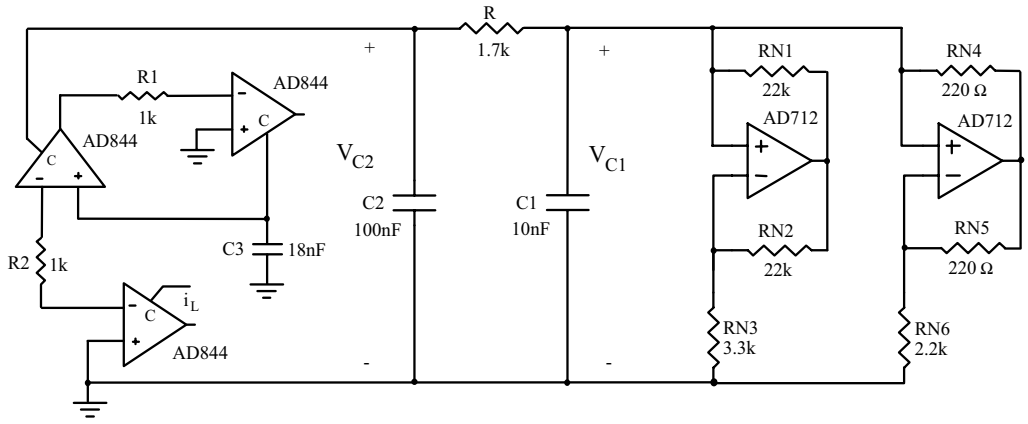


Fig. 11. Hybrid-I realization of Chua's circuit. This realization consists of Kennedy's op amp-based nonlinear resistor and Senani and Gupta's synthetic inductor.

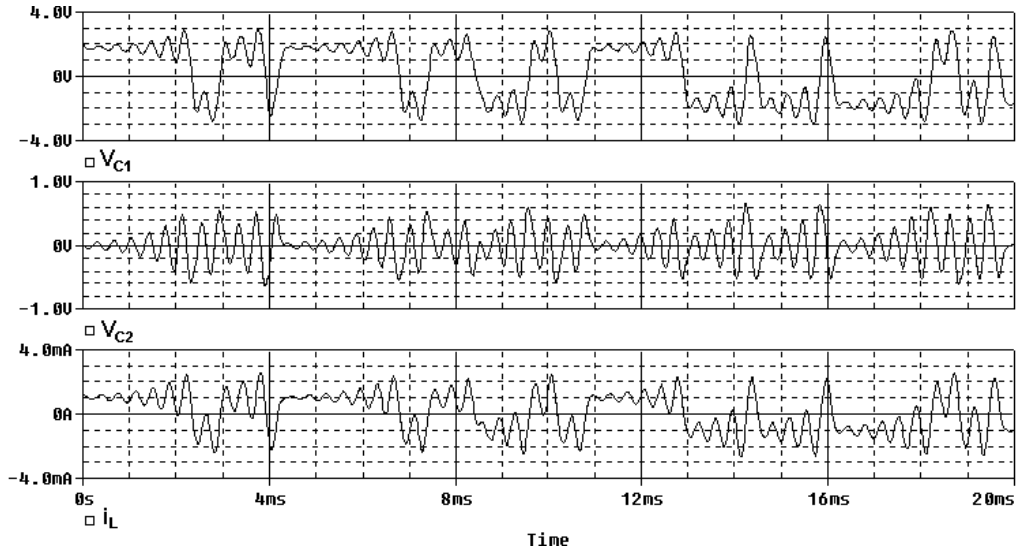
With these element values in [Morgül, 2000], periodic and chaotic oscillations are observed for different values of R in the range $1268\ \Omega - 1837\ \Omega$. The third state variable (inductor current i_L) in Chua's circuit using this topology is not available directly as a current output.

4. Alternative Hybrid Realizations of Chua's Circuit

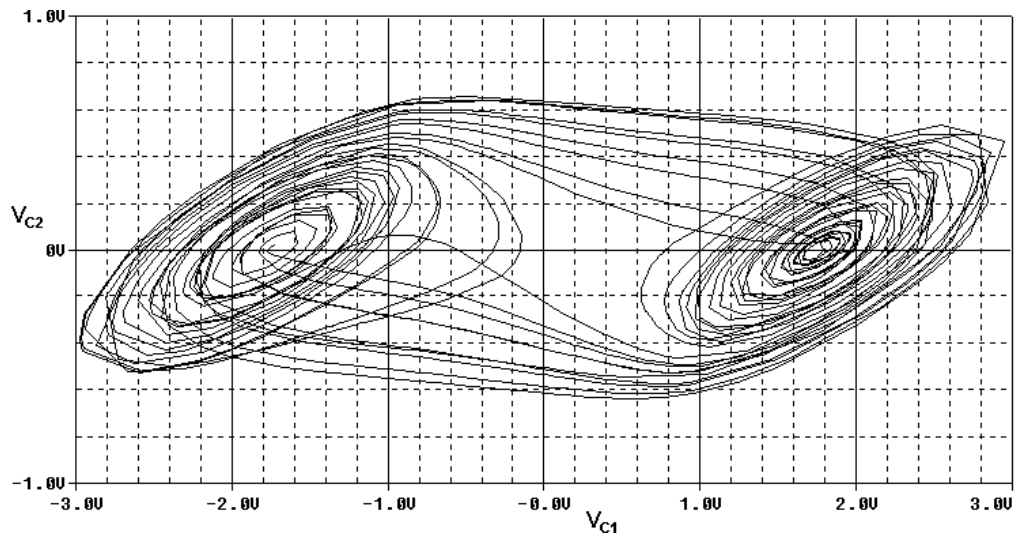
In this section, we present six hybrid realizations of Chua's circuit which exploit the circuit topologies described in Secs. 2 and 3 which have been proposed for Chua's diode and inductor element in Chua's circuit. Our aim is to provide several alternative realizations of Chua's circuit. These novel hybrid realizations are summarized in Table 1. The circuit structure and circuit parameters which yield a double scroll attractor are detailed in the following subsections. The chaotic behaviors of the hybrid realizations are investigated by Pspice simulation experiments.

4.1. *Hybrid-I realization of Chua's circuit*

Figure 11 shows a Hybrid-I realization of Chua's circuit using two AD712 BiFET op amp biased with



(a)



(b)

Fig. 12. (a) Simulations of chaotic circuit dynamics $V_{C1}(t)$, $V_{C2}(t)$ and $i_L(t)$ of Hybrid-I realization (b) The double scroll attractor, projection in the $(V_{C2} - V_{C1})$ plane, in Hybrid-I realization.

± 9 V, and six resistors to implement Kennedy's [1992] VOA-based nonlinear resistor and using three AD844 type CFOA biased with ± 9 V, two resistors, and a capacitor to implement Senani and Gupta's [1998] synthetic inductor for $L = 18$ mH inductance value. While we used the same parameter values for the nonlinear resistor as in [Kennedy, 1992], we configured the parameters of Senani and Gupta's CFOA-based synthetic inductor as $R_1 = R_2 = 1$ kΩ, and $C_3 = 18$ nF to obtain $L = 18$ mH according to Eq. (6). The rest of the parameters are chosen as $C_1 = 10$ nF, $C_2 = 100$ nF, and $R = 1700\ \Omega$ such that the circuit exhibits a double scroll attractor. The PSpice simulation results for the Hybrid-I realization of Chua's circuit are shown in Fig. 12. While the chaotic dynamics V_{C1}, V_{C2} and i_L are shown in Fig. 12(a), the double scroll attractor is observed in Fig. 12(b). This realization uses both VOAs and CFOAs. Due to the use of CFOAs for the synthetic inductor, not only the state variables V_{C1}, and V_{C2}, but the third state variable i_L is also accessible in a direct manner.

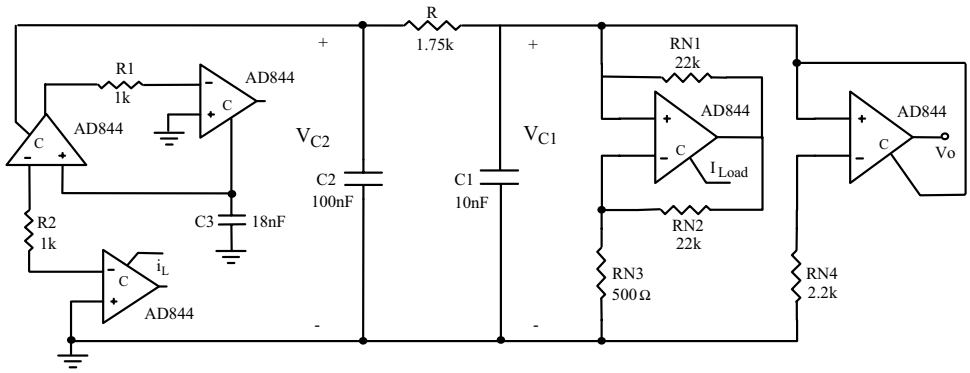
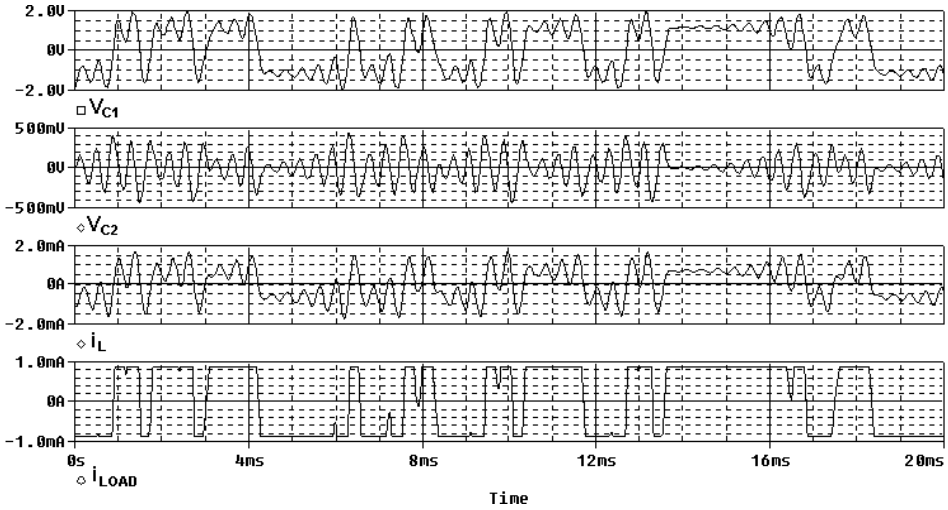
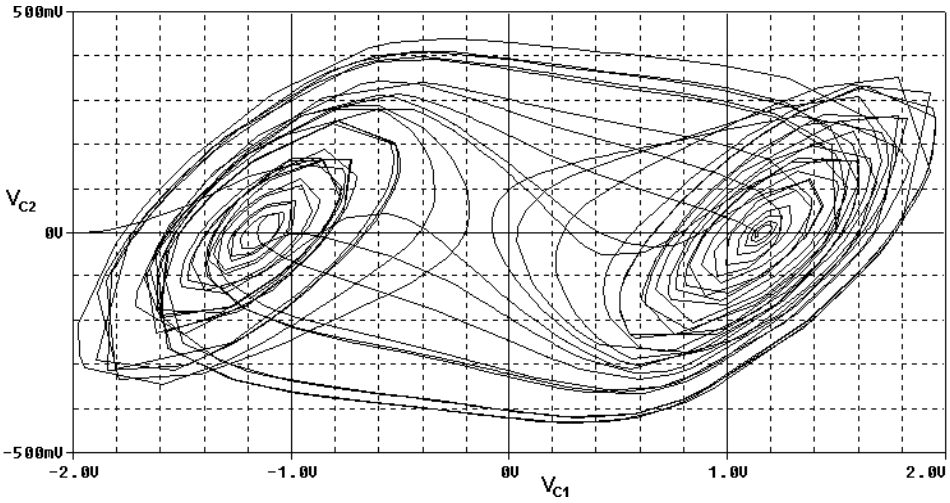


Fig. 13. Hybrid-II realization of Chua's circuit. This realization consists of Elwakil and Kennedy's CFOA-based nonlinear resistor and Senani and Gupta's synthetic inductor.



(a)



(b)

Fig. 14. (a) Simulations of chaotic circuit dynamics $V_{C1}(t)$, $V_{C2}(t)$, $i_L(t)$ and $i_{\mathrm{Load}}(t)$ current output with a 5 kΩ load of Hybrid-II realization. (b) The double scroll attractor, projection in the $(V_{C2} - V_{C1})$ plane, in Hybrid-II realization.

4.2. *Hybrid-II realization of Chua's circuit*

Figure 13 shows a Hybrid-II realization of Chua's circuit using two AD844 type CFOAs biased with ±9 V, and four resistors to implement Elwakil and Kennedy's [2000] nonlinear resistor and using three AD844 type CFOAs biased with ±9 V, two resistors and a capacitor to implement Senani and Gupta's [1998] synthetic inductor with $L = 18$ mH. In this realization, we determined the parameter values of Elwakil and Kennedy's nonlinear resistor as $R_{N1} = R_{N2} = 22$ kΩ, $R_{N3} = 500$ Ω, and $R_{N4} = 2.2$ kΩ as in [Elwakil & Kennedy, 2000], and we used the same parameter values of Senani and Gupta's [1998] synthetic inductor as in our Hybrid-I realization of Chua's circuit with $L = 18$ mH inductance value given by Eq. (6). The rest of the parameters are chosen as $C_1 = 10$ nF, $C_2 = 100$ nF, and $R = 1750$ Ω such that the circuit exhibits a double scroll attractor. The circuit dynamics V_{C1}, V_{C2}, i_L and current output i_{Load} with a 5 kΩ load, and the double scroll attractor obtained from PSpice simulation experiments are shown in Figs. 14(a) and 14(b), respectively. This realization uses only AD844 type CFOA as the active elements. Due to the use of CFOAs for synthetic inductor and nonlinear resistor, all the state variables V_{C1}, V_{C2} and i_L are made available in direct manner. In addition, a buffered and isolated voltage output is also available while the operating frequency is extended to higher frequencies. Moreover, a chaotic current output i_{Load} may be useful in some applications.

4.3. *Hybrid-III realization of Chua's circuit*

Figure 15 shows a Hybrid-III realization of Chua's circuit using two AD712 type op amp biased with ±9 V, and six resistors to implement Kennedy's [1992] nonlinear resistor and using two CMOS-based FTFN blocks, four resistors, and a capacitor to implement an inductance simulator with $L = 18$ mH. In simulation experiments, we used the CMOS realization given in [Çam *et al.*, 2000] for the FTFN blocks. While we determined the parameter values of FTFN-based inductance simulator as $R_1 = R_2 = R_3 = R_4 = 1$ kΩ, and $C_3 = 18$ nF an inductance of $L = 18$ mH according to Eq. (7), we used the same parameter values for the nonlinear resistor as given in [Kennedy, 1992]. The rest of the parameters are chosen as $C_1 = 10$ nF, $C_2 = 100$ nF, and $R = 1625$ Ω such that the circuit exhibits a double scroll attractor. The PSpice simulation results are shown in Fig. 16. Due to the use of a CMOS-based FTFN topology, this realization is suitable for integrated circuit implementation. The FTFN-based inductance simulator used in this realization can also be used for simulating a floating inductance in other chaotic circuits.

4.4. *Hybrid-IV realization of Chua's circuit*

Figure 17 shows the Hybrid-IV realization of Chua's circuit using two AD844 type CFOAs biased with ±9 V, and four resistors to implement Elwakil and Kennedy's [2000] nonlinear resistor and using two FTFN blocks, four resistors, and capacitor to implement an inductance simulator with $L = 18$ mH. In this realization, the nonlinear resistor structure of Hybrid-III is replaced by Elwakil and Kennedy's nonlinear resistor circuit. This reduces the component count and the chaotic spectrum is extended to higher frequencies. In addition, a buffered and isolated voltage output is made available. In both the Hybrid-III and Hybrid-IV realizations, the third

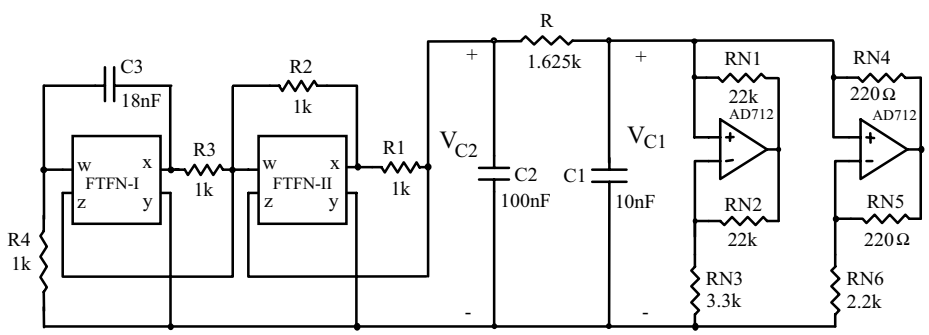
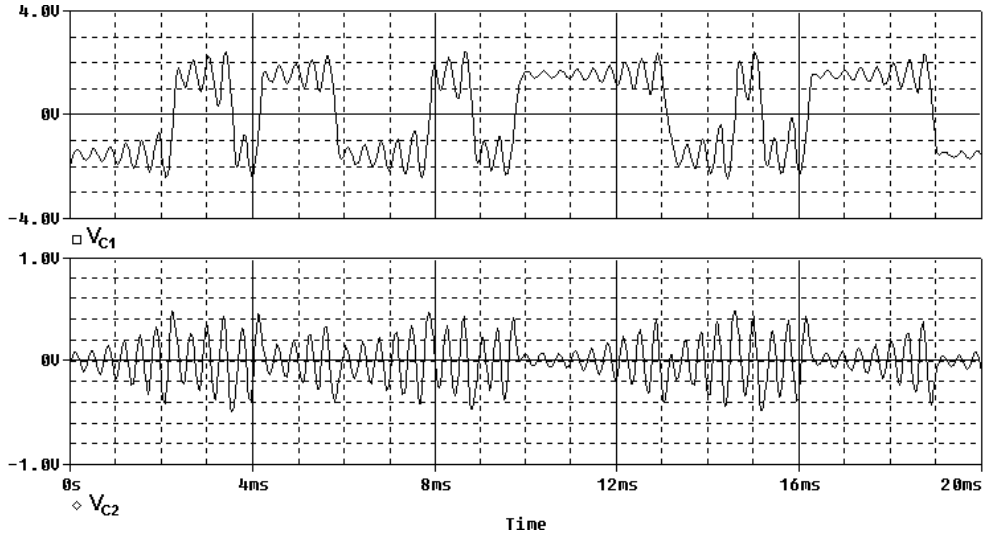
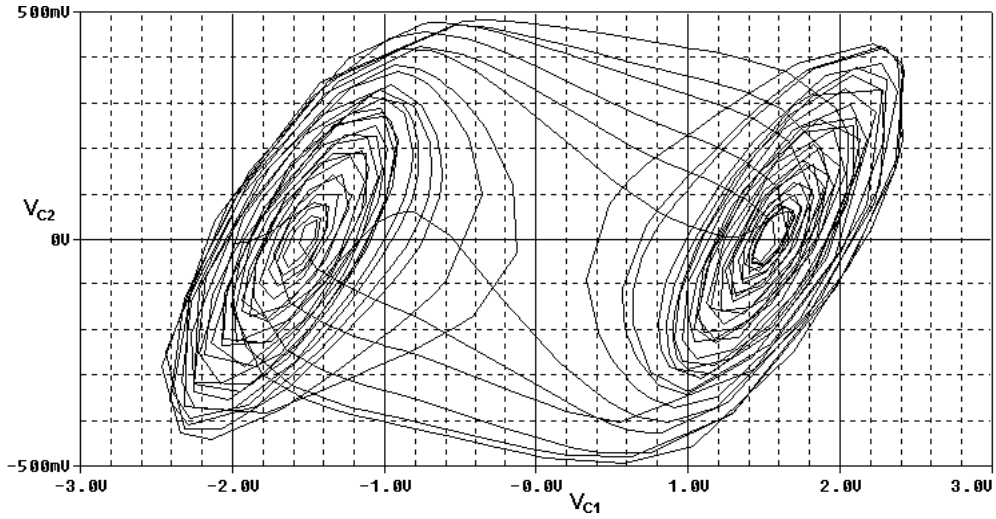


Fig. 15. Hybrid-III realization of Chua's circuit. This realization consists of Kennedy's Op amp-based nonlinear resistor and FTFN-based inductance simulator.



(a)



(b)

Fig. 16. (a) Simulations of chaotic circuit dynamics $V_{C1}(t)$ and $V_{C2}(t)$ of Hybrid-III realization. (b) The double scroll attractor, projection in the $(V_{C2} - V_{C1})$ plane, in Hybrid-III realization.

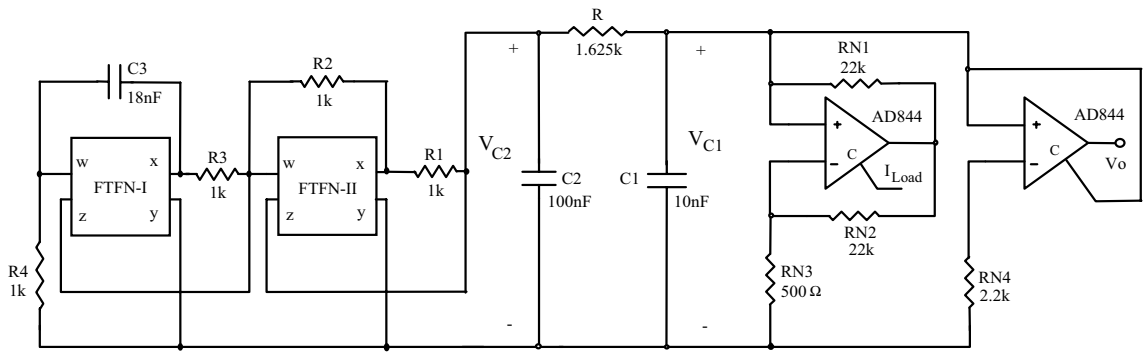
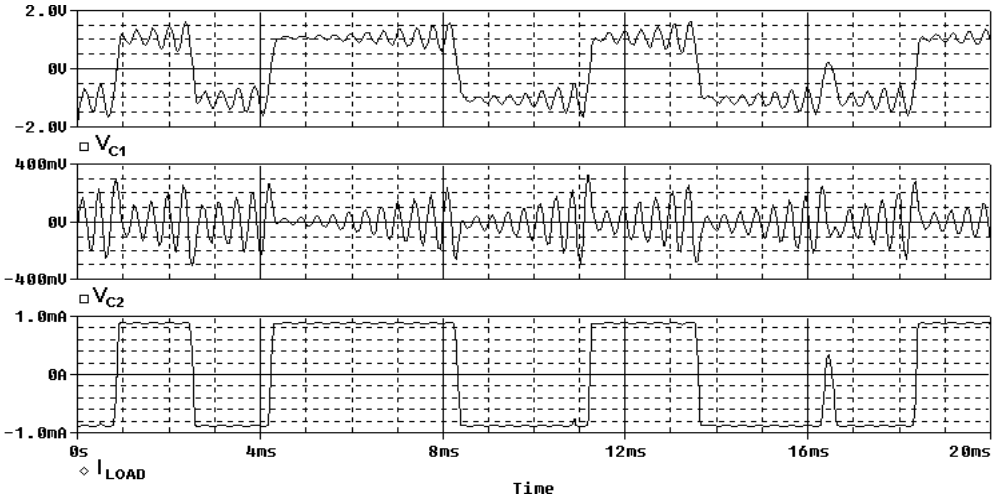
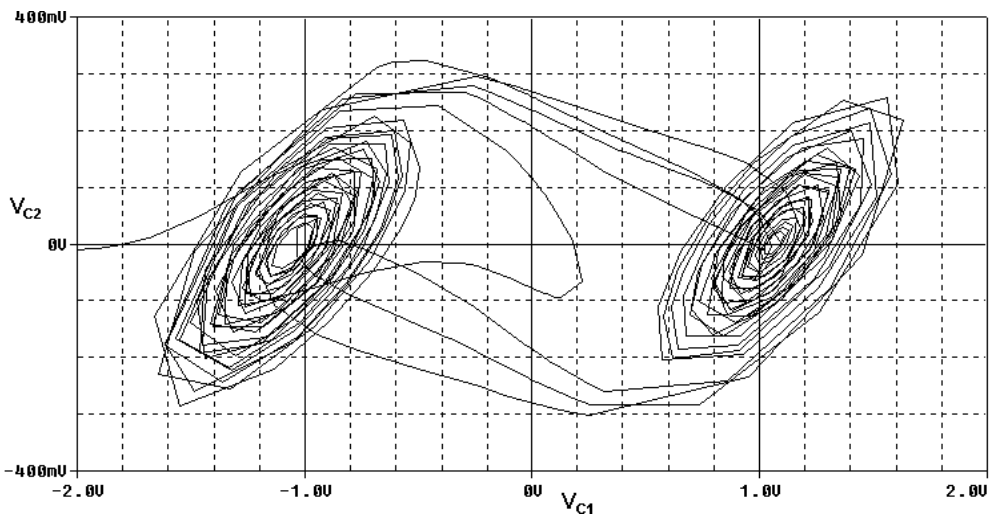


Fig. 17. Hybrid-IV realization of Chua's circuit. This realization consists of Elwakil and Kennedy's CFOA-based nonlinear resistor and FTFN-based inductance simulator.



(a)



(b)

Fig. 18. (a) Simulations of chaotic circuit dynamics $V_{C1}(t)$, $V_{C2}(t)$ and $i_{\text{Load}}(t)$ current output with a 5 kΩ load of Hybrid-IV realization. (b) The double scroll attractor, projection in the $(V_{C2} - V_{C1})$ plane, in Hybrid-IV realization.

variable i_L is not available in a direct manner. The circuit's variables V_{C1}, V_{C2}, i_{Load} (current output with a 5 kΩ load) and a double scroll attractor obtained by Pspice simulations are shown in Figs. 18(a) and 18(b), respectively.

4.5. *Hybrid-V realization of Chua's circuit*

Figure 19 shows a Hybrid-V realization of Chua's circuit using two AD844 type CFOAs biased with ±9 V, and four resistors to implement Elwakil and Kennedy's [2000] nonlinear resistor and using two AD712 type op amp biased with ±9 V, four resistors and a capacitor to implement a VOA-based inductance simulator [Torres & Aguirre, 2000] with $L = 18$ mH. While we used the same parameters for Elwakil and Kennedy's nonlinear resistor as in [Elwakil & Kennedy, 2000], the VOA-based inductance simulator's parameters are determined as $R_1 = R_2 = R_3 = R_4 = 1$ kΩ, and $C_3 = 18$ nF giving $L = 18$ mH according to Eq. (5). The rest of the parameters are chosen as $C_1 = 10$ nF, $C_2 = 100$ nF, and $R = 1650\ \Omega$ such that the circuit exhibits a double scroll attractor. The advantages of using Elwakil and Kennedy's CFOA-based nonlinear resistor are also valid for this realization. The nonidealities and

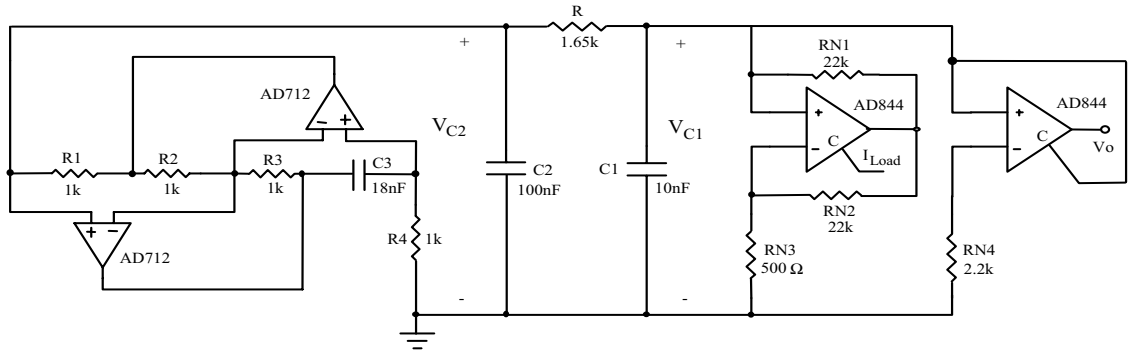
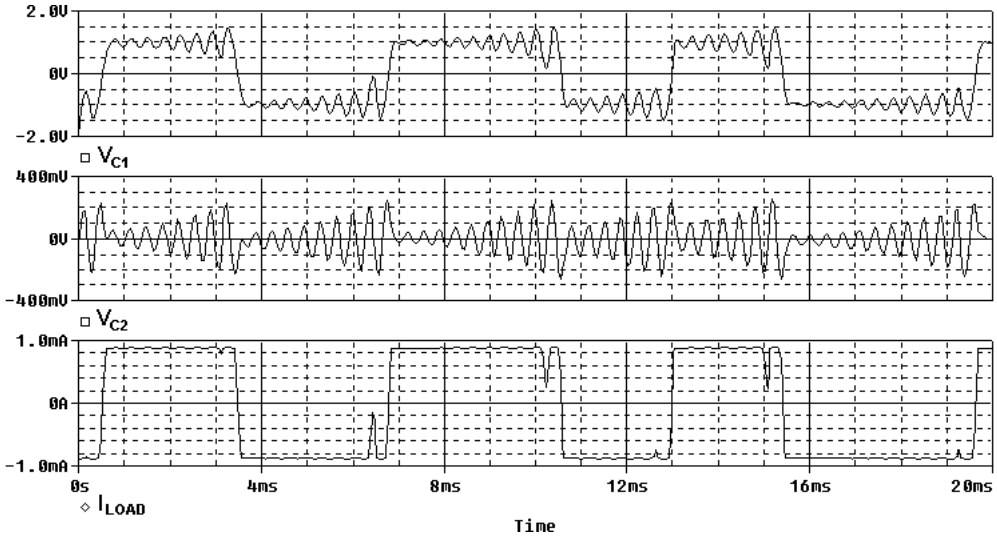
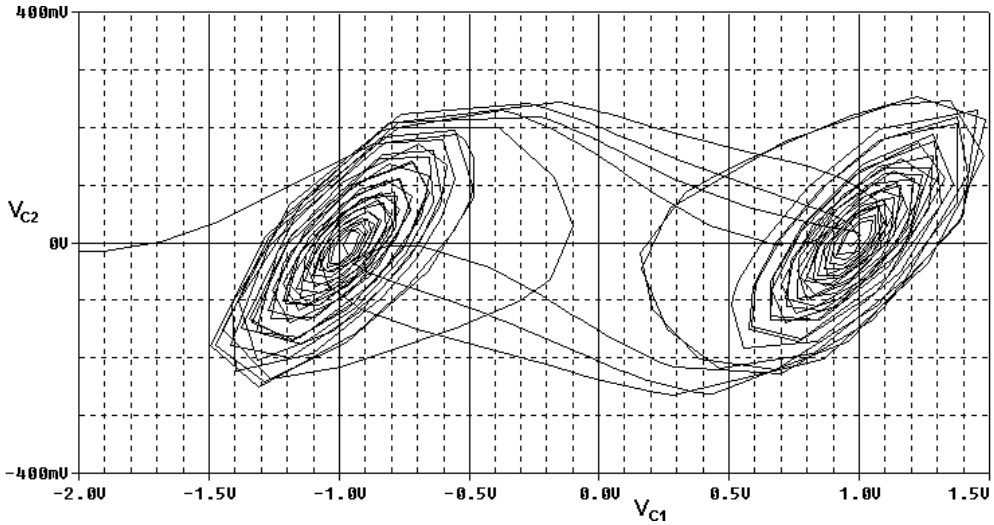


Fig. 19. Hybrid-V realization of Chua's circuit. This realization consists of Elwakil and Kennedy's CFOA-based nonlinear resistor and Op amp-based inductance simulator.



(a)



(b)

Fig. 20. (a) Simulations of chaotic circuit dynamics $V_{C1}(t)$, $V_{C2}(t)$ and $i_{\mathrm{Load}}(t)$ current output with a 5 kΩ load of Hybrid-V realization. (b) The double scroll attractor, projection in the $(V_{C2} - V_{C1})$ plane, in Hybrid-V realization.

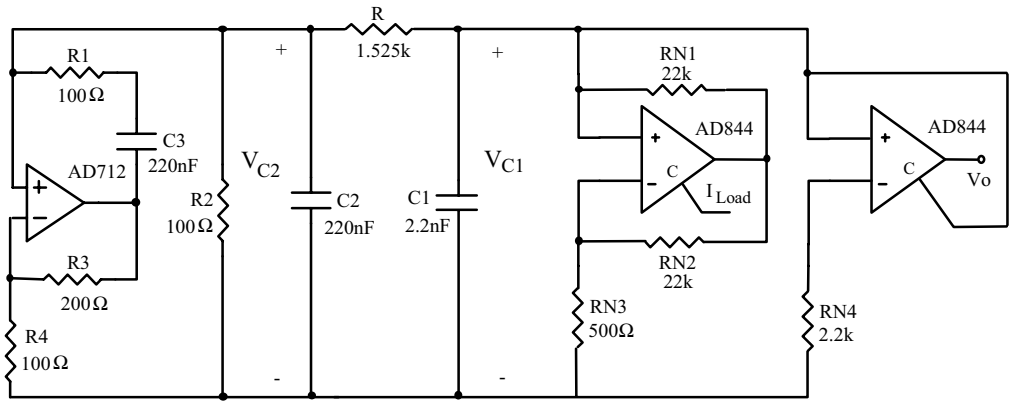
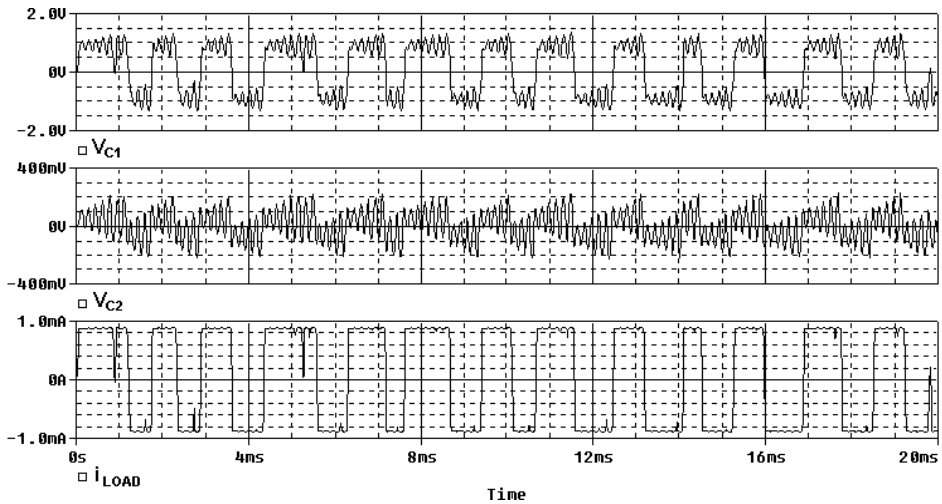
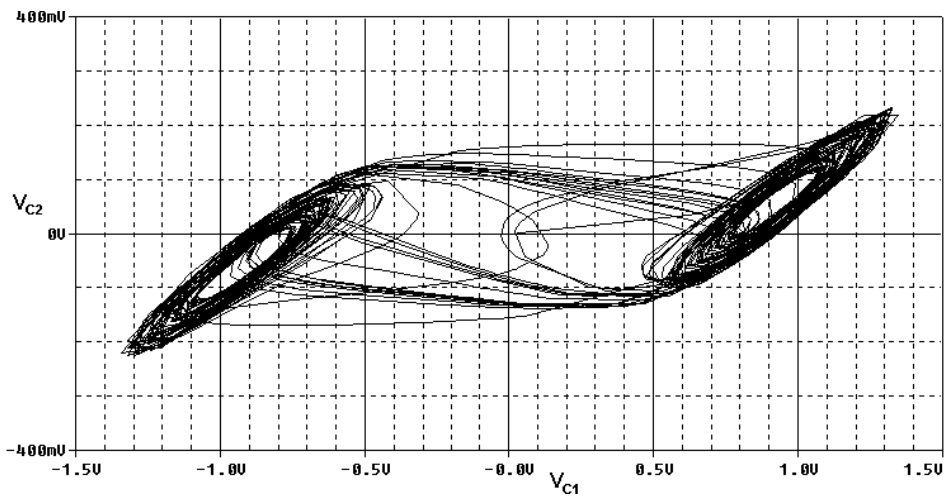


Fig. 21. Hybrid-VI realization of Chua's circuit. This realization consists of Elwakil and Kennedy's CFOA-based nonlinear resistor and Wien bridge-based RC configuration.



(a)



(b)

Fig. 22. (a) Simulations of chaotic circuit dynamics $V_{C1}(t)$, $V_{C2}(t)$ and $i_{\mathrm{Load}}(t)$ current output with a 5 kΩ load of Hybrid-VI realization. (b) The double scroll attractor, projection in the $(V_{C2} - V_{C1})$ plane, in Hybrid-VI realization.

parasitic effects of the VOA should be considered for obtaining the real value of the inductor. The simulation results of this realization are shown in Fig. 20.

4.6. *Hybrid-VI realization of Chua's circuit*

Figure 21 shows a Hybrid-VI realization of Chua's circuit using two AD844 type CFOA biased with ±9 V, and four resistors to implement Elwakil and Kennedy's nonlinear resistor and using an AD712 VOA biased with ±9 V, four resistors and two capacitors to implement Morgül's [2000] Wien bridge-based RC configuration for the LC resonator in Chua's circuit. While we used the same parameters for Elwakil and Kennedy's nonlinear resistor as in [Elwakil & Kennedy, 2000], the parameters of the RC configuration are determined as $R_1 = 100\ \Omega$, $R_2 = 100\ \Omega$, $R_3 = 200\ \Omega$, $R_4 = 100\ \Omega$, and $C_2 = C_3 = 220$ nF. The rest of the circuit parameters are chosen as $C_1 = 2.2$ nF, and $R = 1525\ \Omega$ such that the circuit exhibits a double scroll attractor. The simulation results for this realization are shown in Fig. 22. The advantages of using Elwakil and Kennedy's CFOA-based nonlinear resistor are also valid for this realization. Here, the third state variable i_L is not available in direct manner.

5. Conclusion

Since Chua' circuit exhibits a rich variety of bifurcations and chaos, it has been accepted a paradigm for experimental studies with chaotic dynamics. Several realizations of Chua's circuit have been proposed to date. In this paper, reviewing the existing circuit topologies in the literature for the nonlinear resistor and inductor element in Chua's circuit, we have presented six novel hybrid realizations of Chua's circuit. By using various inductorless topologies, all hybrid realizations presented here may be suitable for integrated circuit implementation. The Hybrid-II realization, which is constructed with Elwakil and Kennedy's CFOA-based nonlinear resistor and Senani and Gupta's CFOA-based synthetic inductor, has some advantages over the other hybrid realizations. In the Hybrid-II realization, due to the use of CFOA-based topologies for both the nonlinear resistor and inductor elements, all the state variables V_{C1}, V_{C2} and i_L are made available in a direct manner. In addition, a buffered and isolated voltage output is also available while the operating frequency is extended. Moreover, a chaotic current output i_{Load} may be useful in some applications. The Pspice simulation results confirm that these hybrid realizations exhibit the original chaotic behavior of Chua's circuit. We hope that these hybrid realizations will provide alternative solutions to the reader studying Chua's circuit.

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