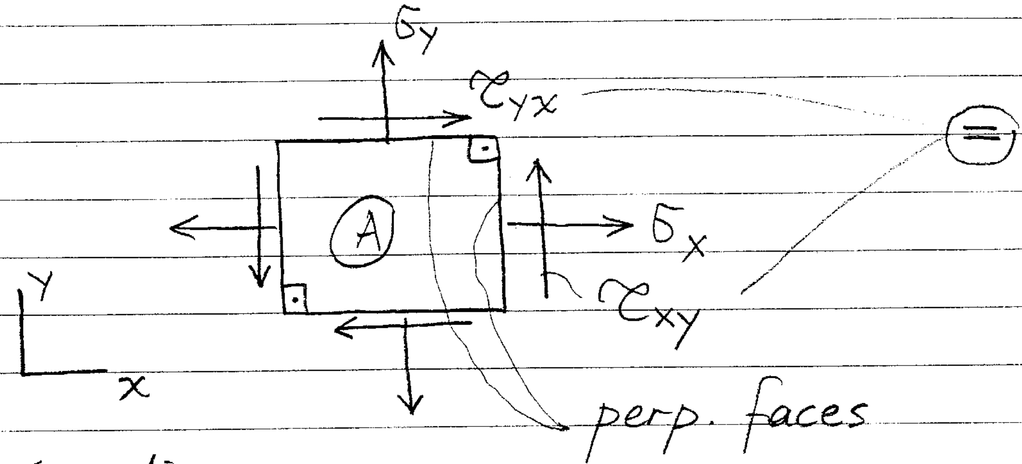
FBD for (A):



$$\underbrace{\overbrace{\sigma_x}^{\text{in } x\text{-dir.}}, \overbrace{\sigma_y}^{\text{in } y\text{-dir.}}}_{\substack{\oplus \text{ if directed outwards (T)} \\ \ominus \text{ if directed inwards (C)}}} \rightarrow \text{normal stresses (NS)}$$

⊕ if directed outwards (T)
⊖ " " inwards (C)

$\tau_{\underbrace{x}_{\text{face}}\underbrace{y}_{\text{dir.}}} \rightarrow$ shear stress (SS) acting on x-f in y-dir.

$\tau_{yx} \rightarrow$ SS acting on y-f in x-dir.

$\left\{\begin{array}{l} \tau_{xy} \text{ is } \oplus \text{ if it acts in } \oplus\text{-}y \text{ dir.} \\ \tau_{yx} \text{ " " " " " " } x \text{ "} \end{array}\right.$

This sign convention holds for ⊕-f's.

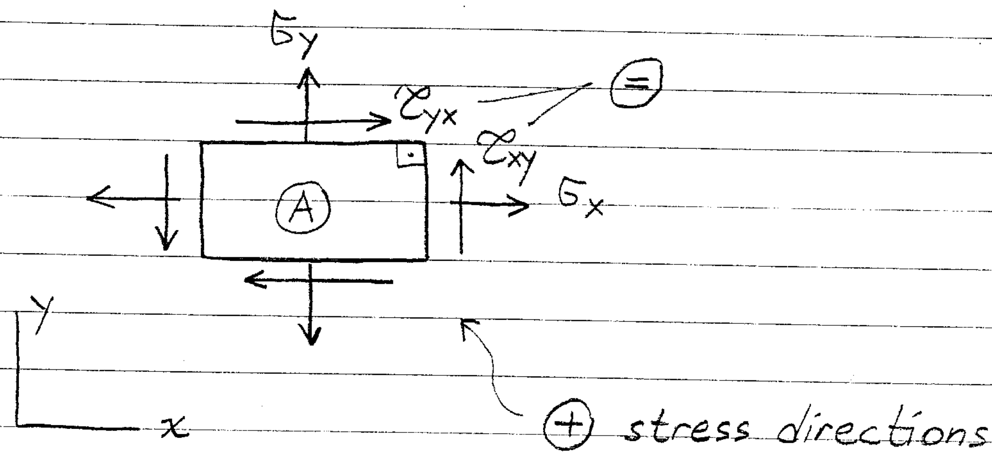
$\Rightarrow$ The opposite sign convention holds for ⊖-f's.

$$\boxed{\tau_{xy} = \tau_{yx}}$$

Stress components:

$\left.\begin{matrix}\sigma_x\\ \sigma_y\end{matrix}\right)$ NS $\qquad$ $\left.\begin{matrix}\tau_{xy}\\ \tau_{yx}\end{matrix}\right)$ SS

=



we can express the stress components in matrix form as follows:

$$\underline{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}$$

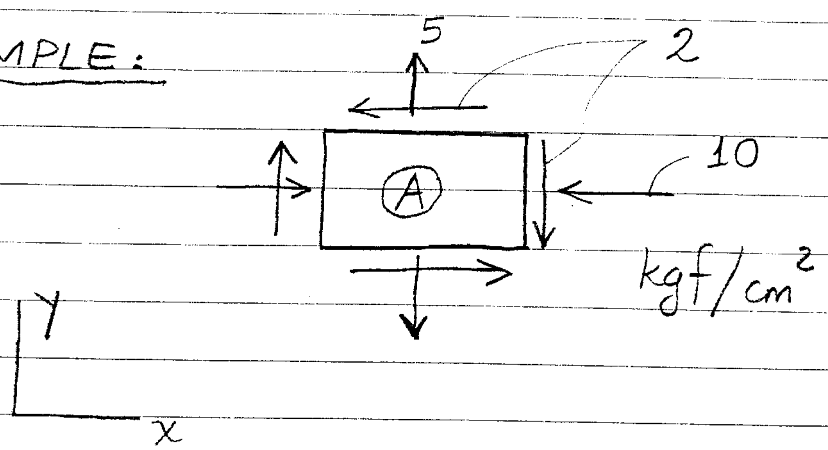
stress comp.'s of x-f.

stress comp's of y-f.

stress matrix

(2x2) symmetric matrix

EXAMPLE:

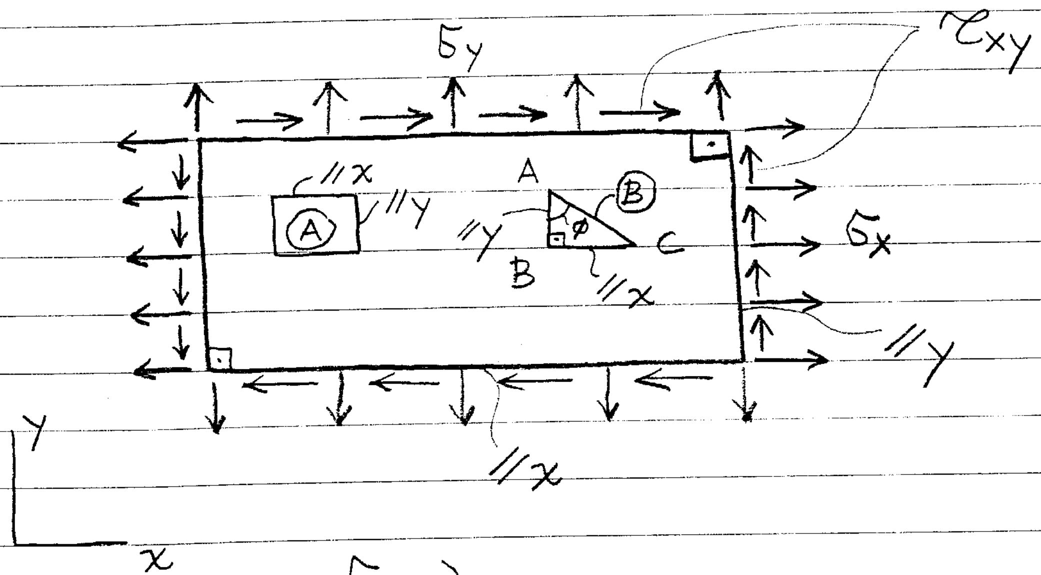


$$\Rightarrow \quad \sigma_x = -10 \quad \text{C}$$
$$\sigma_y = +5 \quad \text{T}$$

$$\tau_{xy} = -2$$

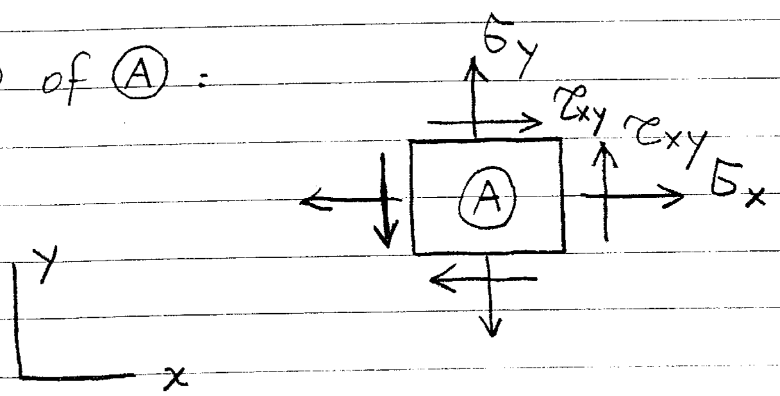
$$\Rightarrow \quad \underline{\sigma} = \begin{bmatrix} -10 & -2 \\ -2 & 5 \end{bmatrix}$$

Consider a rectangular plate whose sides are subjected to uniform stresses:

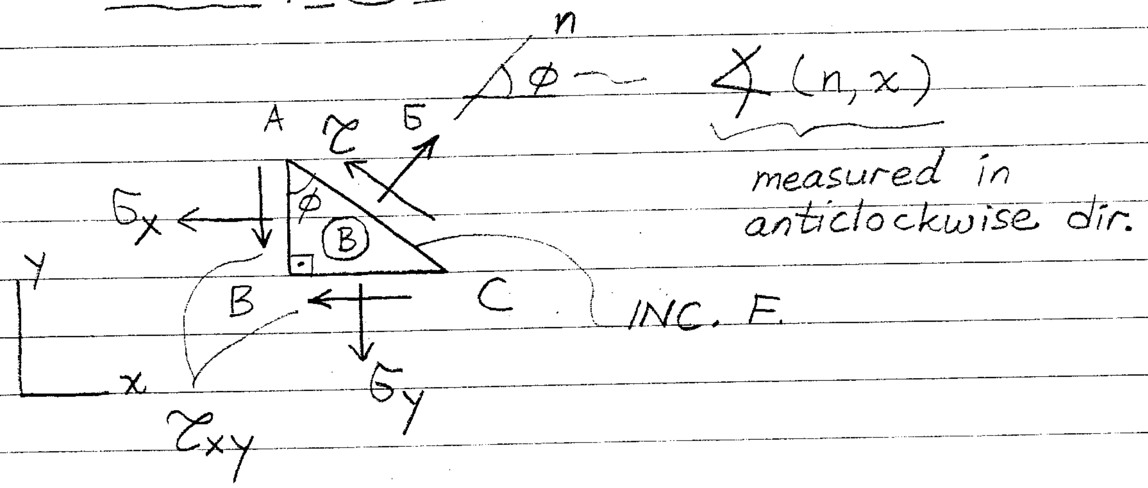


$$\left.\begin{array}{l} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{array}\right\} \text{GIVEN} \Rightarrow \text{KNOWN}$$

FBD of A:



FBD of (B) :



wish to find σ, τ in terms of σ_x, σ_y and τ_{xy}. For that use EE :

$t \rightarrow$ thickness of the plate

$\overset{+}{\nearrow} \sum F_\sigma = 0 \leftarrow$ EE in σ-dir

$$\Rightarrow \quad \sigma * \overline{AC} * t - \sigma_y \sin\phi * \overline{BC} * t$$

$$(*) \quad - \sigma_x \cos\phi * \overline{AB} * t - \tau_{xy} \cos\phi * \overline{BC} * t$$

$$- \tau_{xy} \sin\phi * \overline{AB} * t = 0$$

$(*) / \overline{AC}$:

$$\Rightarrow \quad \boxed{\sigma = \sigma_y \sin^2\phi + \sigma_x \cos^2\phi + 2\tau_{xy} \sin\phi \cos\phi} \quad (**)$$

$$\left.\begin{aligned} \sin^2\phi &= \frac{1}{2}(1-\cos 2\phi) \\ \cos^2\phi &= \frac{1}{2}(1+\cos 2\phi) \\ \sin\phi\cos\phi &= \frac{1}{2}\sin 2\phi \end{aligned}\right\} \text{A}$$

A $\rightarrow$ (**) :

$$\Rightarrow \boxed{\sigma = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi} \quad (1)$$

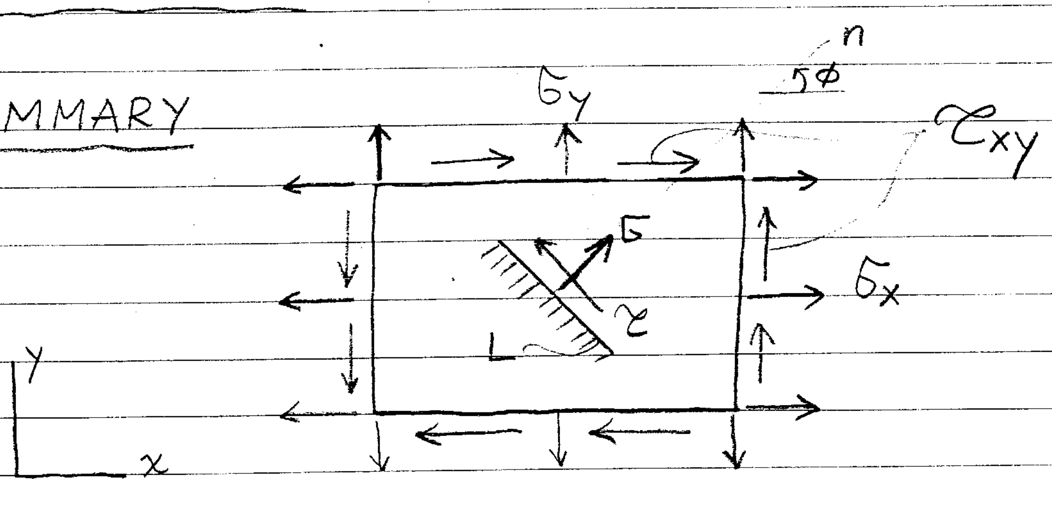
Now write:

$\oplus \nwarrow \sum F_\tau = 0 \leftarrow$ EE in τ - dir.

$$\Rightarrow \boxed{\tau = -\frac{\sigma_x-\sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi} \quad (2)$$

(1) & (2) determine σ & τ in terms of σ_x, σ_y, τ_{xy}

SUMMARY



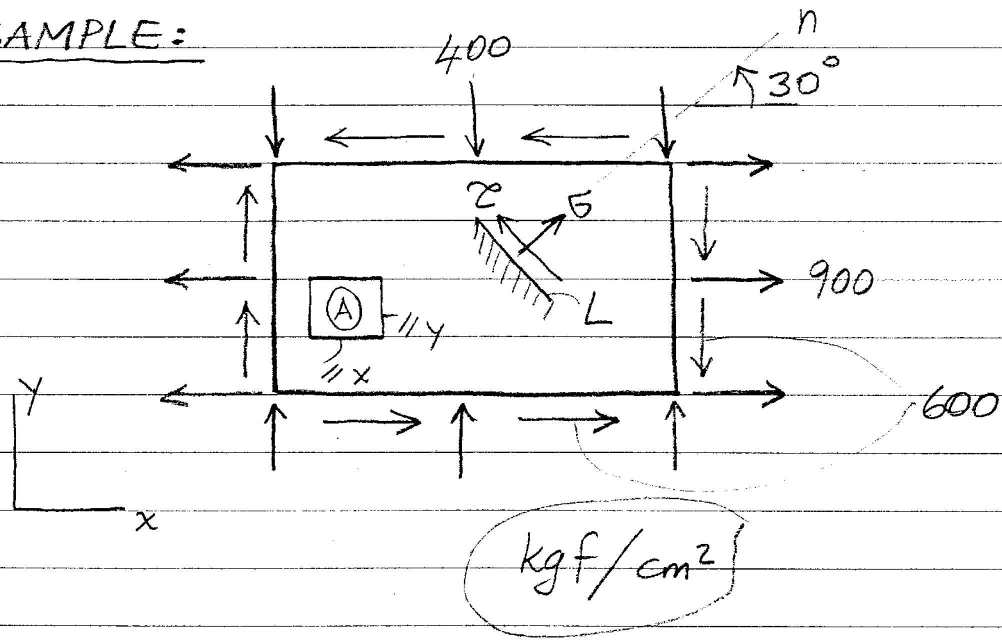
$\sigma_x, \sigma_y, \tau_{xy} \rightarrow$ GIVEN

σ, τ acting on L:

$$\boxed{\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\phi + \tau_{xy} \sin 2\phi} \quad (1)$$

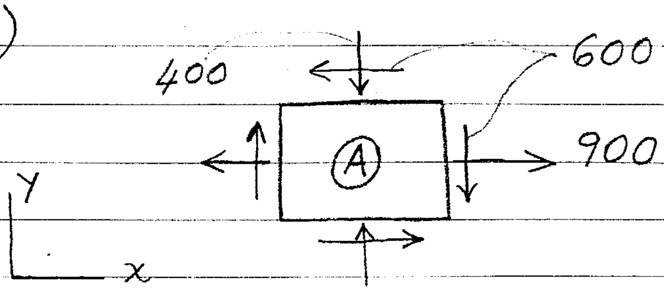
$$\boxed{\tau = -\frac{\sigma_x - \sigma_y}{2} \sin 2\phi + \tau_{xy} \cos 2\phi} \quad (2)$$

EXAMPLE:



we wish to determine the stresses acting
a) on A
b) on L

a)



b) $\left.\begin{array}{l}\sigma_x = 900 \\ \sigma_y = -400 \\ \tau_{xy} = -600\end{array}\right\}$ use 1 and 2

$$\phi = 30^\circ \Rightarrow 2\phi = 60^\circ$$

$$\Rightarrow \cos 2\phi = \frac{1}{2}, \quad \sin 2\phi = \frac{\sqrt{3}}{2}$$

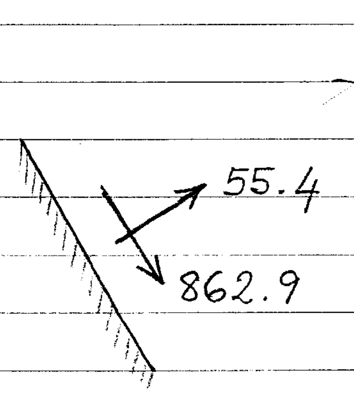
$$\Rightarrow \frac{\sigma_x + \sigma_y}{2} = 250, \quad \frac{\sigma_x - \sigma_y}{2} = 650$$

$$1 \Rightarrow \sigma = 250 + (650) * \frac{1}{2} + (-600) * \frac{\sqrt{3}}{2}$$

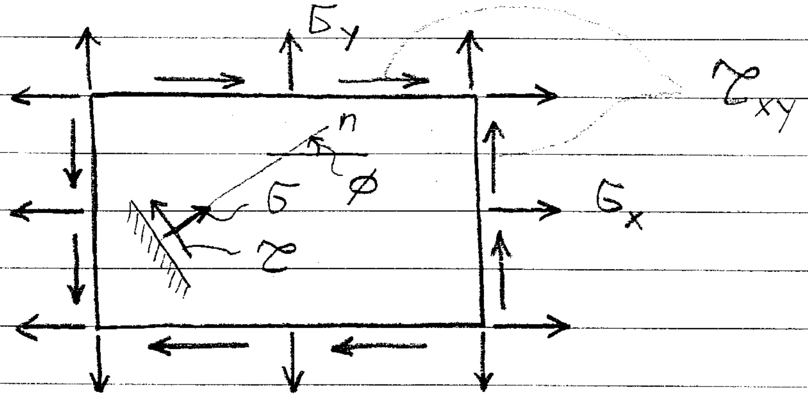
$$\Rightarrow \sigma \cong 55.4 \text{ kgf/cm}^2 \ (T)$$

$$2 \Rightarrow \tau = -(650) * \frac{\sqrt{3}}{2} + (-600) * \frac{1}{2}$$

$$\Rightarrow \tau = -862.9 \text{ kgf/cm}^2$$



MC FOR PLANE STATE OF STRESS

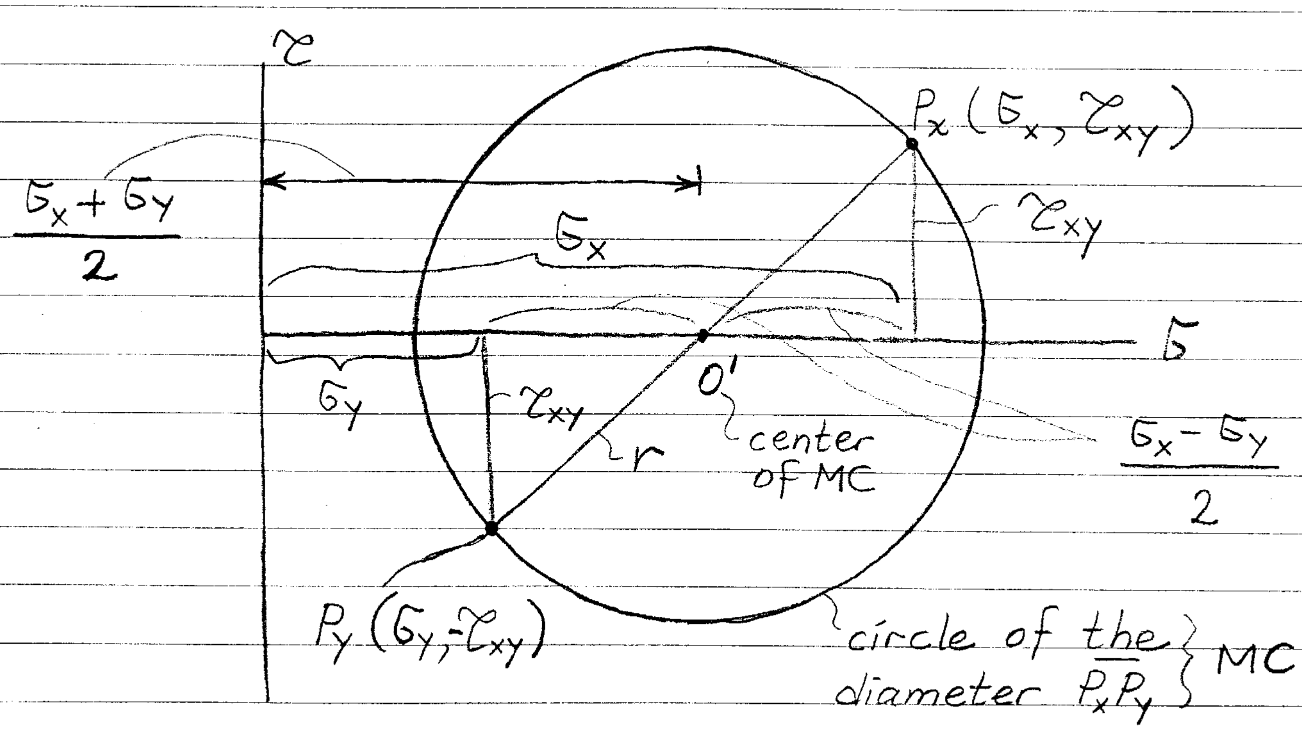


$$\left.\begin{matrix}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{matrix}\right\} \text{GIVEN}$$

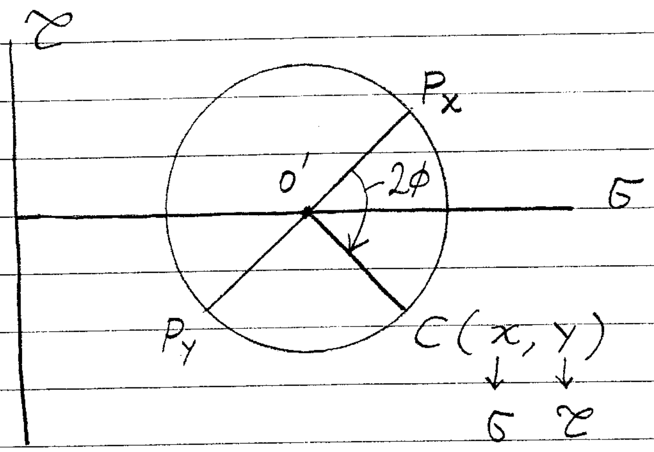
wish to find σ and τ by MC.

For that use the following steps.

1) Construct MC:



2) Take the angle 2ϕ from $\overline{O'P_x}$ in clockwise dir.



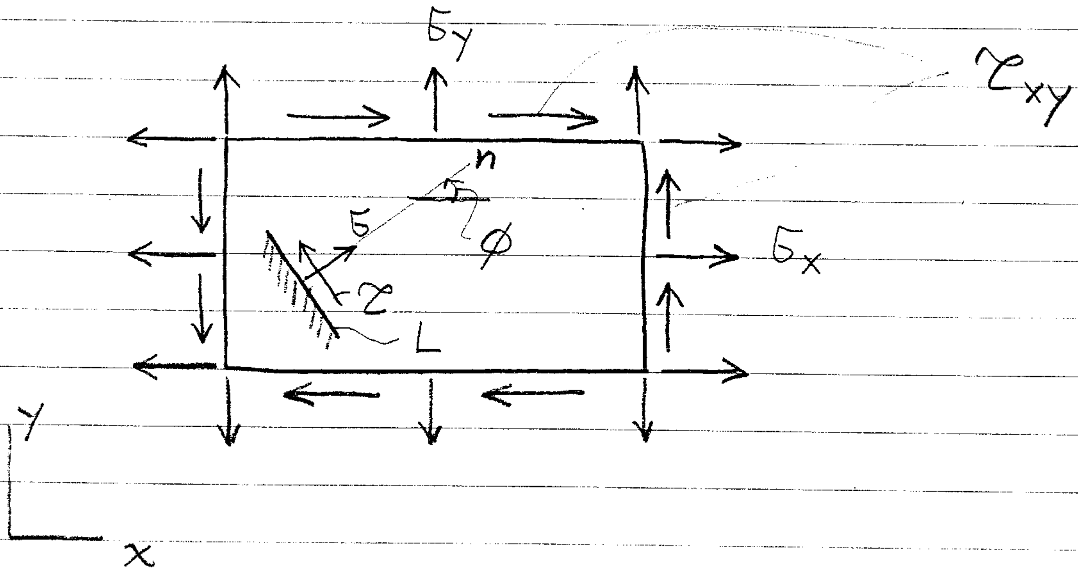
This determines a pt "C" on MC

3) by computing the coordinates of C find σ and τ acting on L

$$C \rightarrow (\underset{\substack{\downarrow \\ \sigma}}{x}, \underset{\substack{\downarrow \\ \tau}}{y})$$

PRINCIPAL STRESSES

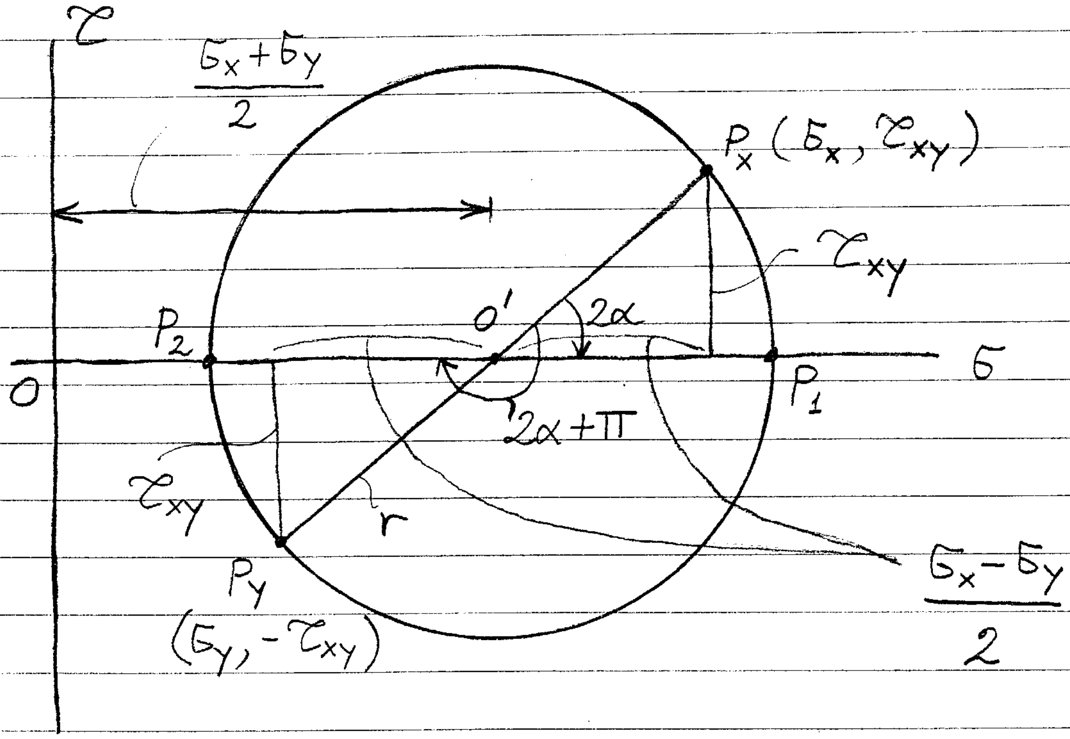
- The plane on which $\tau = 0$ is called principal plane – PP
- The normal stresses acting on PP are called principal stresses – PS
- The element formed by PP's is called principal element – PE



$$\left.\begin{array}{l}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{array}\right\} \text{KNOWN}$$

wish to find PS's.

Use MC :



$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\alpha = \frac{\tau_{xy}}{\frac{\sigma_x-\sigma_y}{2}} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sin 2\alpha = \frac{\tau_{xy}}{r}$$

these two eqs. determine "α" uniquely

At P_1 and P_2 $\tau = 0$

$\Rightarrow P_1, P_2$ will determine PP's and PS's

P_1 :

$$2\phi = 2\alpha \Rightarrow \phi = \alpha$$

$$\Rightarrow \tau = 0$$

$$\sigma = \overline{OO'} + r \qquad \frac{\sigma_x + \sigma_y}{2}$$

$$\Rightarrow \boxed{\sigma = \frac{\sigma_x + \sigma_y}{2} + r} \quad \sigma_1$$

P_2 :

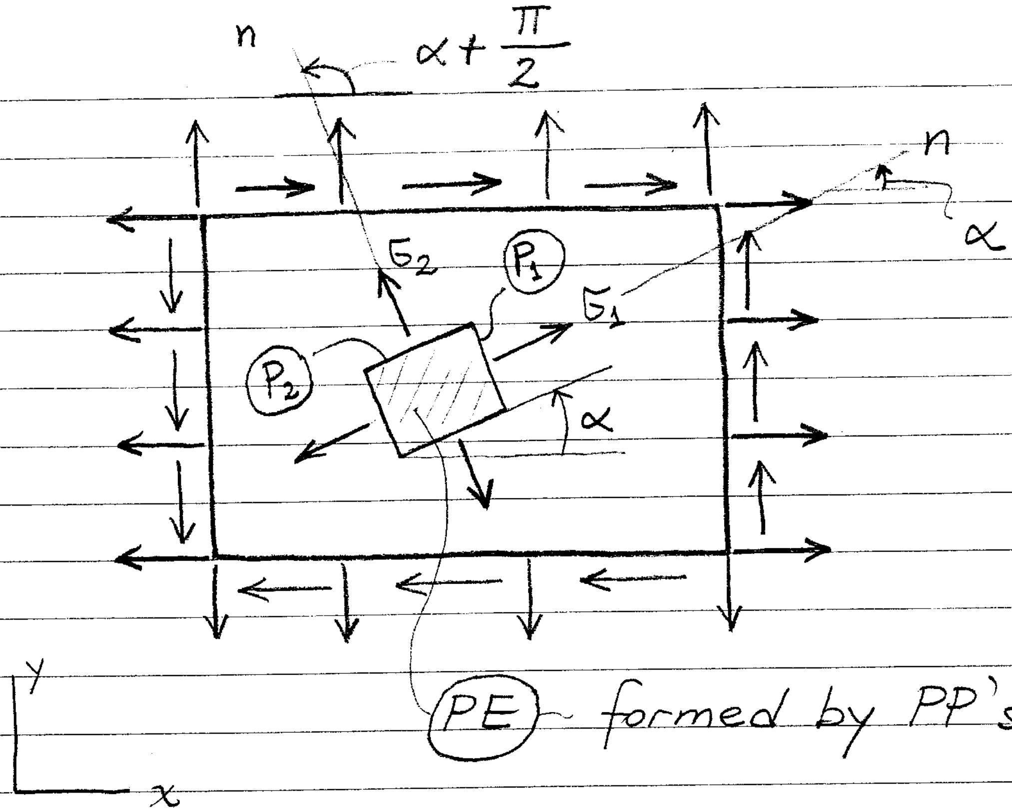
$$2\phi = 2\alpha + \pi \Rightarrow \phi = \alpha + \frac{\pi}{2}$$

$$\Rightarrow \tau = 0$$

$$\sigma = \overline{OO'} - r \qquad \frac{\sigma_x + \sigma_y}{2}$$

$$\Rightarrow \boxed{\sigma = \frac{\sigma_x + \sigma_y}{2} - r}$$

$$\sigma_2$$



$\sigma_1, \sigma_2 \rightarrow$ PS's

SUMMARY

To compute PE and PS's use the following steps:

1) Compute $r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

2) Compute the angle α from:

$$\operatorname{Tan} 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}, \quad \operatorname{Sin} 2\alpha = \frac{\tau_{xy}}{r}$$

$$\Rightarrow \alpha$$

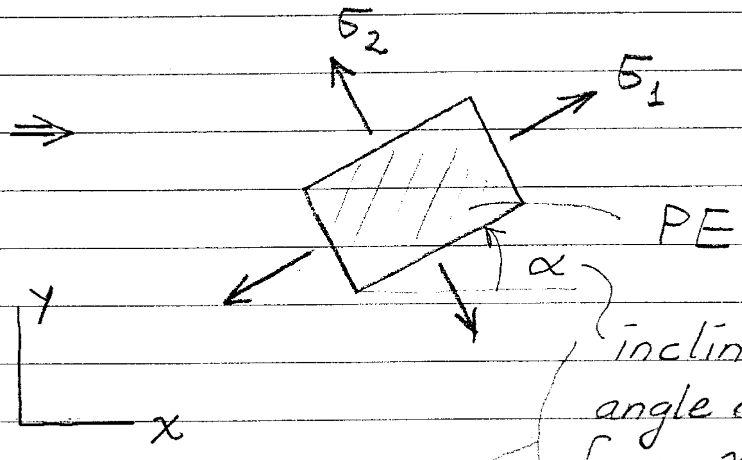
$\rightarrow$

3) Compute PS's by:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + r$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - r$$

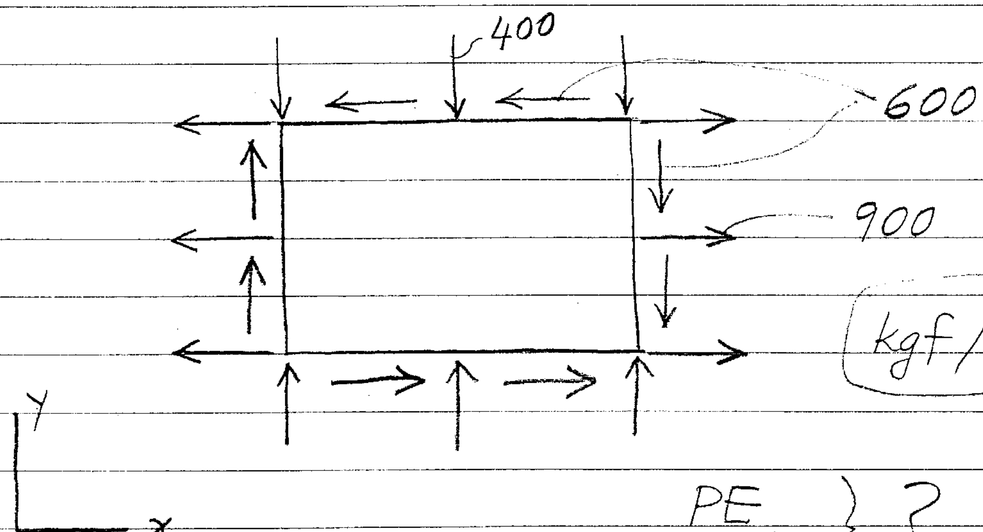
$\Rightarrow$



α inclination angle of PE from x-axis

measured in anticlockwise dir.

EXAMPLE: (Previous example)



kgf/cm^2

PE
PS's } ?

$\sigma_x = 900$ (T)

$\sigma_y = -400$ (C)

$\tau_{xy} = -600$

1) $r = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\Rightarrow \boxed{r \cong 884.6 \ kgf/cm^2}$

2) $\tan 2\alpha = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{2 * (-600)}{1300}$

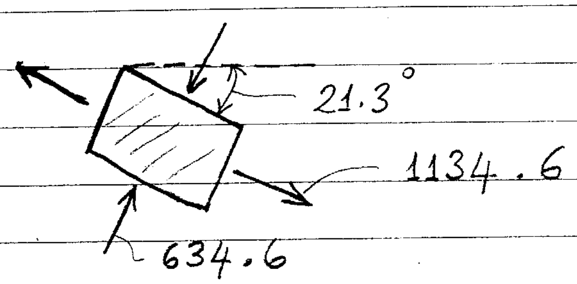
$\Rightarrow \tan 2\alpha \cong -0.92 \Rightarrow 2\alpha = -42.6^\circ \cancel{+\pi}$?

$\sin 2\alpha = \dfrac{\overset{-}{\tau_{xy}}}{\underset{+}{r}} < 0$

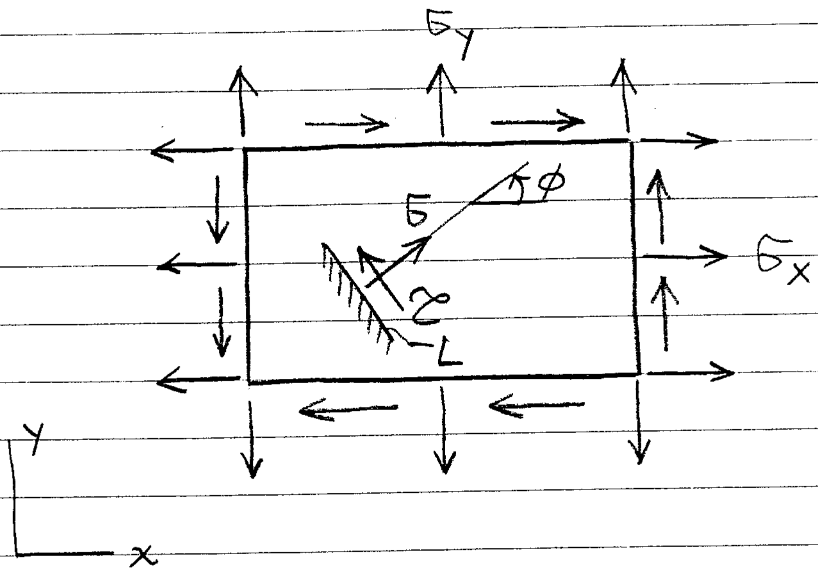
$\Rightarrow \boxed{\alpha = -21.3^\circ}$

3) $\sigma_1 = \overbrace{\dfrac{\sigma_x + \sigma_y}{2}}^{250} + \underbrace{r}_{884.6} = 1134.6$ (T)

$\sigma_2 = \dfrac{\sigma_x + \sigma_y}{2} - r = -634.6$ (C)

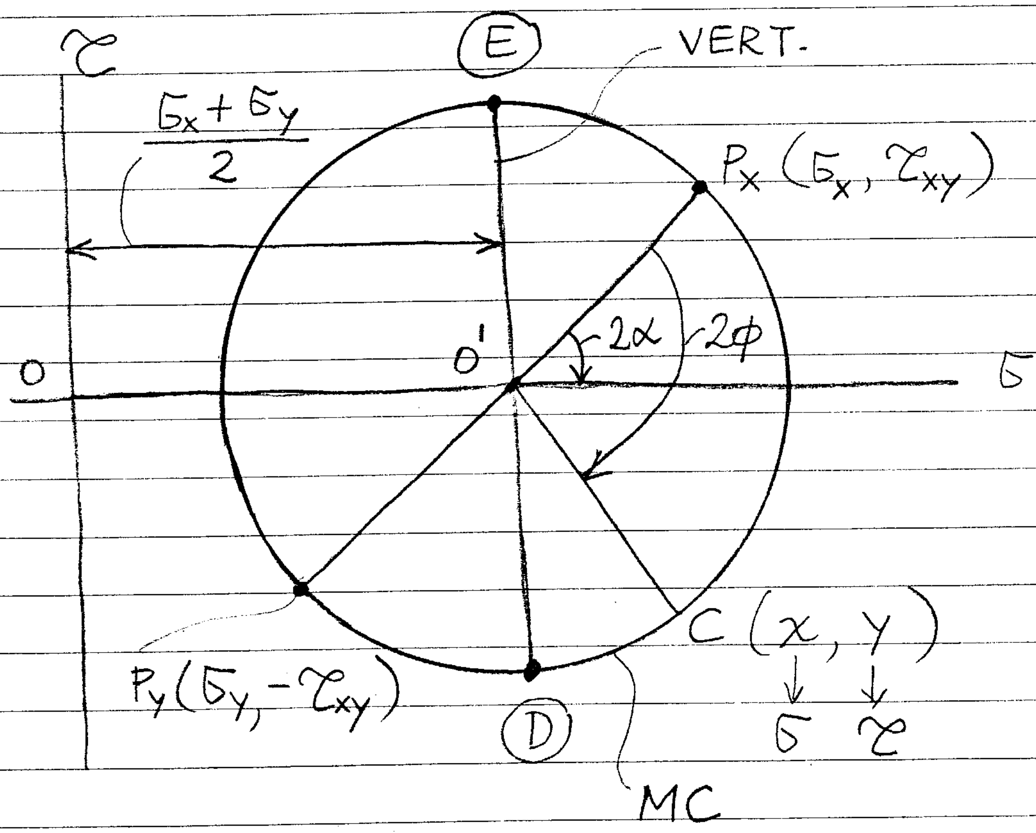


τ_{max} FOR PLANE STATE OF STRESS



$\sigma_x, \sigma_y, \tau_{xy} \rightarrow$ GIVEN

Construct MC:



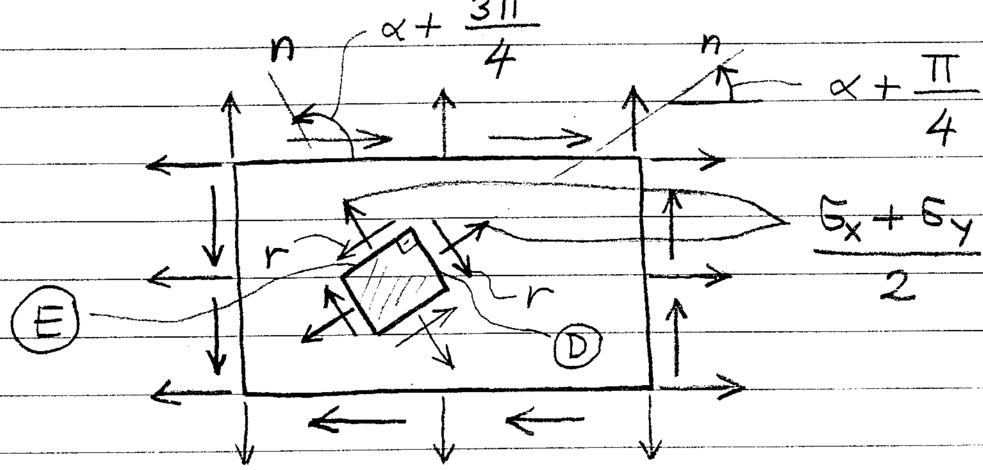
$\Rightarrow \tau$ will be max at D & E

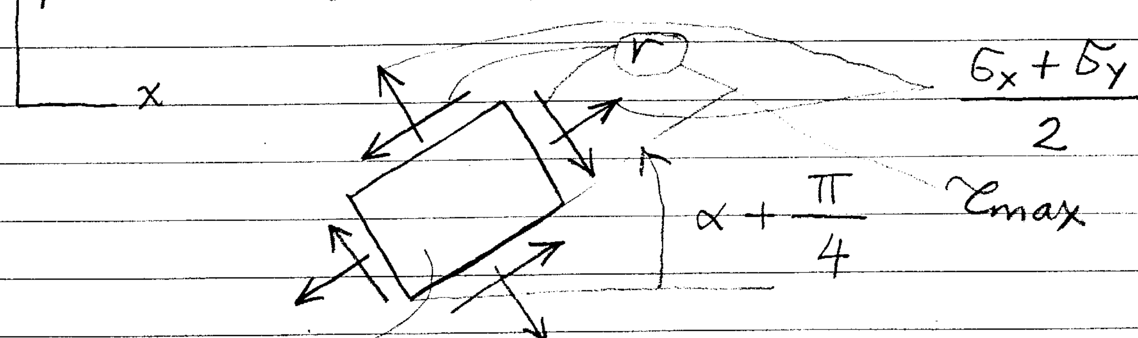
$\Rightarrow \tau_{max}$ will occur at D & E

$$\tau_{max} = \max|\tau|$$

$$\Rightarrow \boxed{\tau_{max} = r} \quad \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

D	E
$2\phi = 2\alpha + \frac{\pi}{2}$	$2\phi = 2\alpha + \frac{3\pi}{2}$
$\Rightarrow \phi = \alpha + \frac{\pi}{4}$	$\Rightarrow \phi = \alpha + \frac{3\pi}{4}$
$\sigma = \frac{\sigma_x + \sigma_y}{2}$	$\sigma = \frac{\sigma_x + \sigma_y}{2}$
$\tau = -r$	$\tau = +r$





The element on which τ_{max} acts.

EXAMPLE: (Previous example)

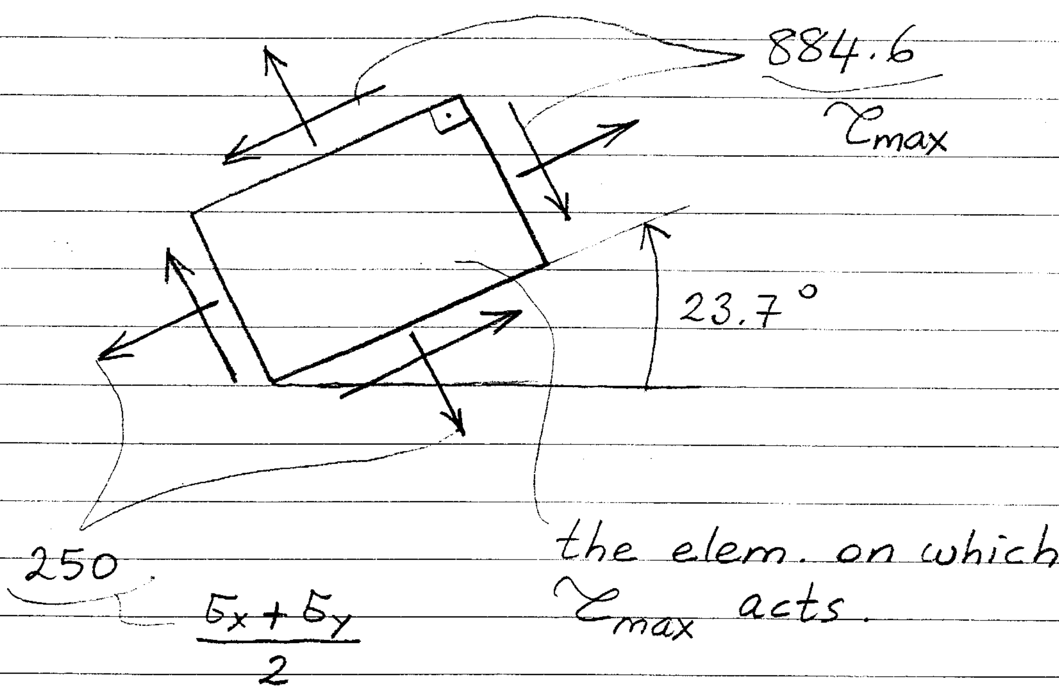
$$\sigma_x = 900$$
$$\sigma_y = -400 \qquad \text{kgf/cm}^2$$
$$\tau_{xy} = -600$$

τ_{max} and the elem. on which it acts?

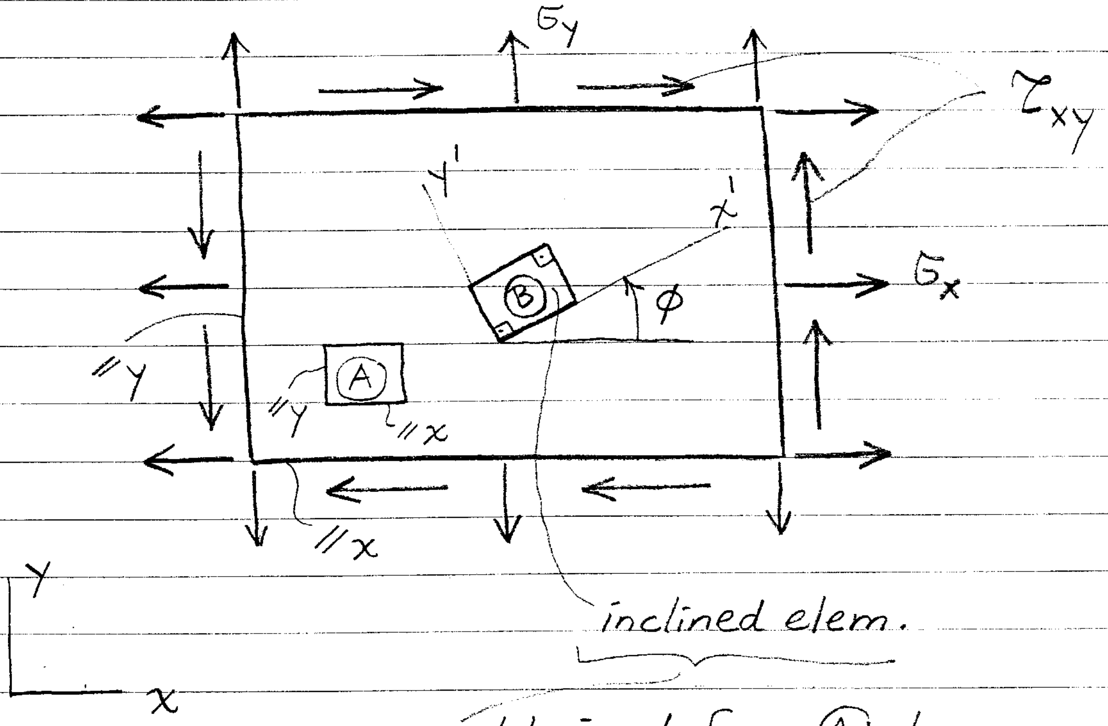
$$\left.\begin{aligned} r &= 884.6 \ \text{kgf/cm}^2 \\ \alpha &= -21.3^\circ \end{aligned}\right\} \text{computed previously}$$

$$\Rightarrow \quad \boxed{\tau_{max} = r = 884.6 \ \text{kgf/cm}^2}$$

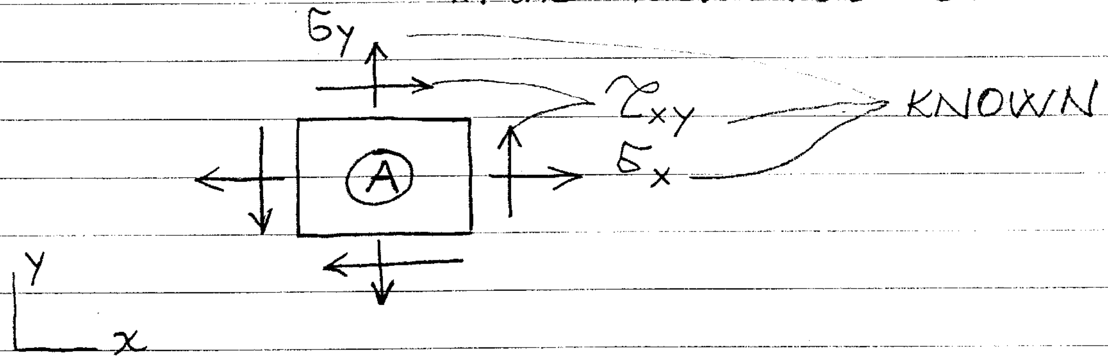
$$\underbrace{\alpha}_{-21.3^\circ} + \underbrace{\frac{\pi}{4}}_{45^\circ} = 23.7^\circ$$



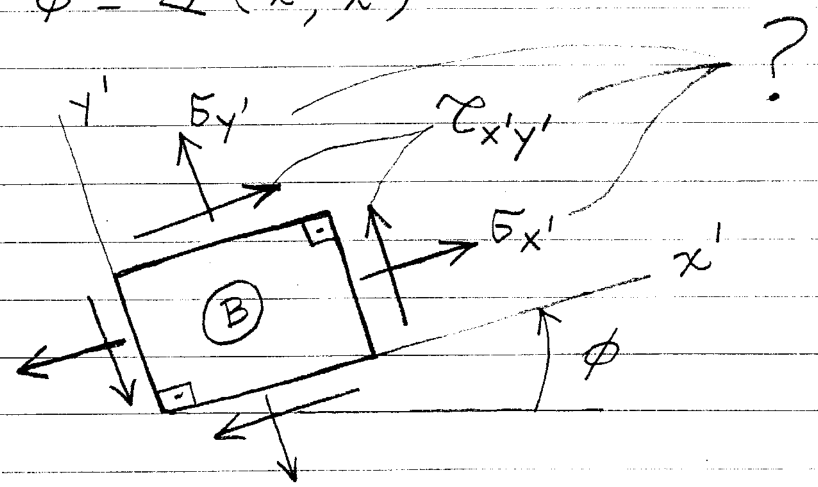
TRANSFORMATION FORMULAS



obtained from (A) by rotating it by the angle ϕ in the anticlockwise dir.



$$\phi = \measuredangle (x', x)$$



We wish to find $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$ in terms of known σ_x, σ_y, τ_{xy}

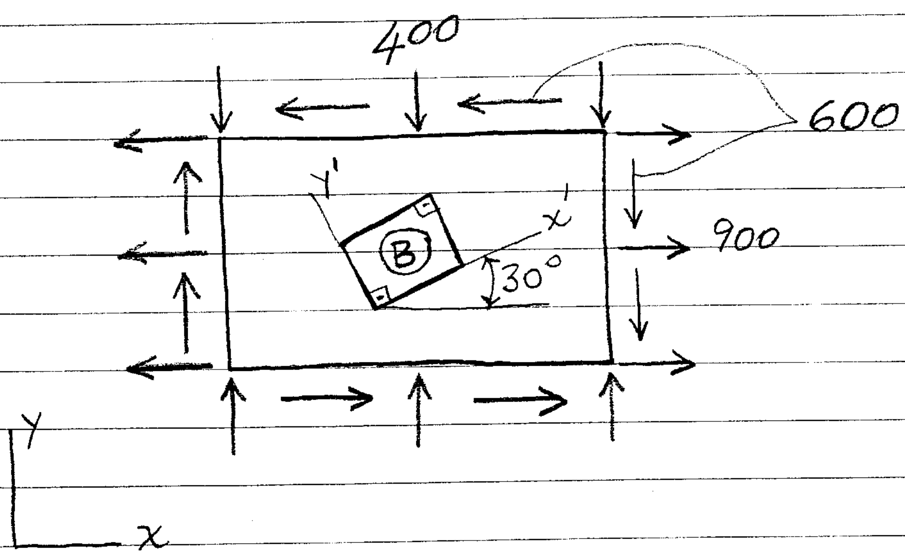
They can be found by using the following transformation formulas:

$$\textcircled{1}\quad \sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\phi + \tau_{xy} \sin 2\phi$$

$$\textcircled{2}\quad \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\phi - \tau_{xy} \sin 2\phi$$

$$\textcircled{3}\quad \tau_{x'y'} = -\frac{(\sigma_x - \sigma_y)}{2} \sin 2\phi + \tau_{xy} \cos 2\phi$$

EXAMPLE: (Previous example)



we wish to find the stresses acting on (B).

$$\phi = \measuredangle(x', x) = 30^\circ \Rightarrow 2\phi = 60^\circ$$

$$\Rightarrow \sin 2\phi = \frac{\sqrt{3}}{2}, \quad \cos 2\phi = \frac{1}{2}$$

$$\left.\begin{aligned} \sigma_x &= 900 \\ \sigma_y &= -400 \\ \tau_{xy} &= -600 \end{aligned}\right\} \text{kgf/cm}^2$$

$$\frac{\sigma_x + \sigma_y}{2} = 250, \quad \frac{\sigma_x - \sigma_y}{2} = 650$$

$$1 \Rightarrow \sigma_{x'} = (250) + (650)(1/2) + (-600)(\sqrt{3}/2)$$

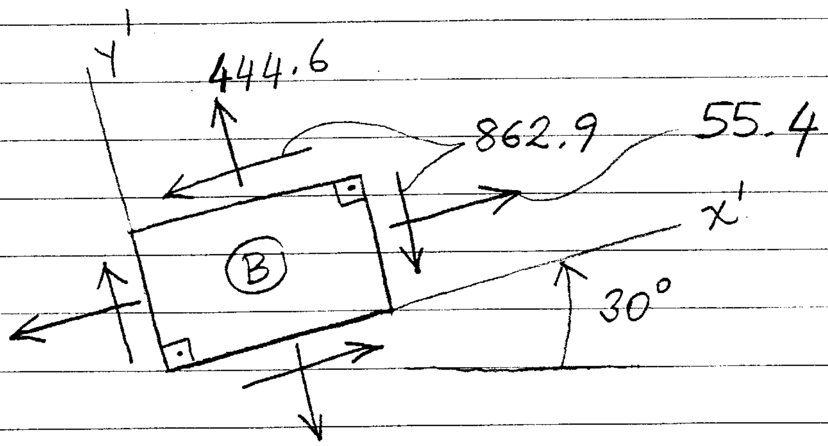
$$\Rightarrow \sigma_{x'} = 55.4 \ \text{T}$$

$$2 \Rightarrow \sigma_{y'} = (250) - (650)(1/2) - (-600)(\sqrt{3}/2)$$

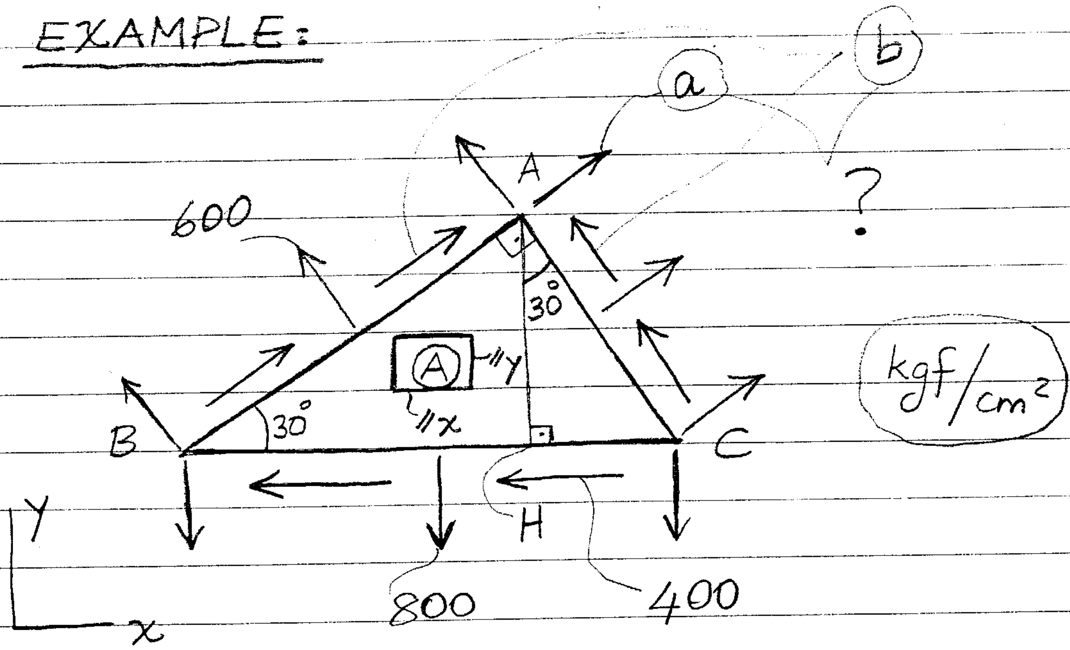
$$\Rightarrow \sigma_{y'} = 444.6 \ \text{T}$$

$$3 \Rightarrow \tau_{x'y'} = -(650)(\sqrt{3}/2) + (-600)(1/2)$$

$$\Rightarrow \tau_{x'y'} = -862.9$$



EXAMPLE:



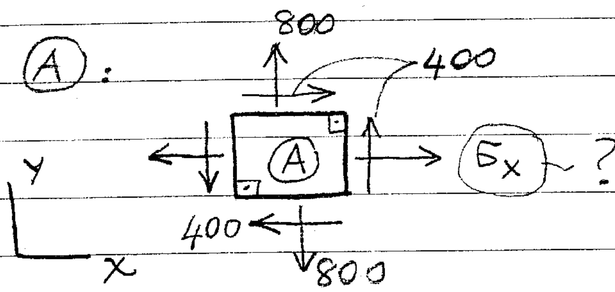
We wish to compute:

a) Stresses acting on A & the stresses "a" and "b"

b) PS's & PE

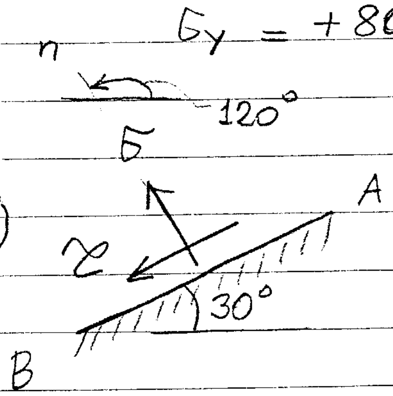
a) 1st method: Use formulas:

FBD of A:



$\Rightarrow$ $\sigma_x = ?$

$\sigma_y = +800$, $\tau_{xy} = +400$



(**)

$\sigma = 600$

$\tau = -b$?

Write the formula for σ: