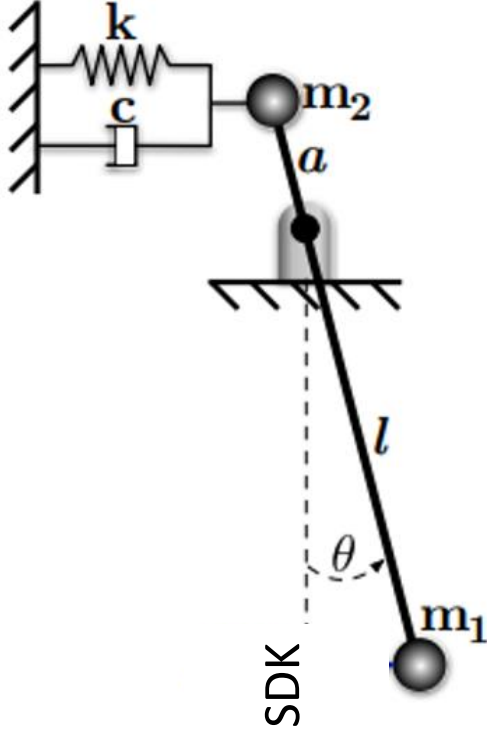


Şekilde kütlesi ihmal edilen çubuğun O noktası etrafında küçük genlikli titreşim yaptığı görülmektedir. Şekildeki parametreleri dikkate alarak;

a) Sistemin hareket denklemini bulunuz.

b) Sistemin sönümsüz serbest titreşiminin doğal frekansını bulunuz.



a)

$$\sum M_0 = -m_1 g L \theta + m_2 g a \theta - c a^2 \dot{\theta} - k a^2 \theta = I_0 \ddot{\theta}$$

$$I_0 \ddot{\theta} + m_1 g L \theta - m_2 g a \theta + c a^2 \dot{\theta} + k a^2 \theta = 0$$

$$I_0 = m_1 L^2 + m_2 a^2$$

$$(m_1 L^2 + m_2 a^2) \ddot{\theta} + c a^2 \dot{\theta} + (m_1 g L - m_2 g a + k a^2) \theta = 0$$

$$\ddot{\theta} + \left(\frac{c a^2}{m_1 L^2 + m_2 a^2} \right) \dot{\theta} + \left(\frac{m_1 g L - m_2 g a + k a^2}{m_1 L^2 + m_2 a^2} \right) \theta = 0$$

b)

$$\omega_n = \sqrt{\frac{m_1 g L - m_2 g a + k a^2}{m_1 L^2 + m_2 a^2}} \quad 1/s$$

LAGRANGE METODU İLE ÇÖZÜM

$$KE = \frac{1}{2} m_1 (L \dot{\theta})^2 + \frac{1}{2} m_2 (a \dot{\theta})^2$$

$$KE = \frac{1}{2} (m_1 L^2 + m_2 a^2) \dot{\theta}^2$$

$$PE = \frac{1}{2} k a^2 \theta^2 - m_2 g a (1 - \cos \theta) + m_1 g L (1 - \cos \theta)$$

$$DE = \frac{1}{2} c a^2 \dot{\theta}^2$$

$$\frac{d}{dt} \frac{\partial KE}{\partial \dot{q}_i} - \frac{\partial KE}{\partial q_i} + \frac{\partial PE}{\partial q_i} + \frac{\partial DE}{\partial \dot{q}_i} = Q(t)_i$$

$$\frac{d}{dt} \frac{\partial KE}{\partial \dot{\theta}} = (m_1 L^2 + m_2 a^2) \ddot{\theta}$$

$$\frac{\partial PE}{\partial \theta} = k a^2 \theta - m_2 g a \theta + m_1 g L \theta$$

$$\frac{\partial DE}{\partial \dot{\theta}} = c a^2 \dot{\theta}$$

a)

$$(m_1 L^2 + m_2 a^2) \ddot{\theta} + ca^2 \dot{\theta} + (m_1 gL - m_2 ga + ka^2) \theta = 0$$

$$\ddot{\theta} + \left(\frac{ca^2}{m_1 L^2 + m_2 a^2} \right) \dot{\theta} + \left(\frac{m_1 gL - m_2 ga + ka^2}{m_1 L^2 + m_2 a^2} \right) \theta = 0$$

b)

$$\omega_n = \sqrt{\frac{m_1 gL - m_2 ga + ka^2}{m_1 L^2 + m_2 a^2}} \quad 1/s$$