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Performance Evaluation of a Two-stage Supply Chain Under Fuzzy Environment

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In this paper, performance evaluation of a two-stage supply chain which consists of a supplier and a retailer under uncertainty environment is examined. In the literature; many studies have been investigated on supply chain performance analysis in uncertainty environment and demand parameter is an essential data on these analyses. When there is sufficient data about demand, it is easy to construct probability distribution of demand. However, in the absence of enough historical data, it is impossible to construct the probability distribution. The demand parameter is intuitively determined and shown by a fuzzy distribution in such situations. This paper intends to find closed form solutions in order to maximize the performance of a two-stage supply chain with coordination under fuzzy environment. The coordination of the supply chain is maintained by contracts. The contracts analysed in this study are revenue sharing contract and buy back contract. Numerical examples are presented to show the validity of the proposed models and to investigate the effect of contract parameters on the total performance of the supply chain.

Keywords: Supply chain; contracts; fuzzy demand; fuzzy optimization; revenue sharing; buyback

1 INTRODUCTION

A supply chain is a system of all the organizations, activities, resources and technology involved in the production and sale of raw materials from the sup-

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plier to the retailer. In the supply chain, there are many members, such as suppliers, retailers, distributors, etc. These members usually act as independent members. To improve total supply chain performance, they should coordinate with each other and make decisions to improve overall supply chain performance. Supply chains are classified as centralized and decentralized supply chain model by decision maker. In a centralized supply chain, there is a decision maker who wants to optimize the performance of the entire supply chain. However, sometimes this situation cannot be achieved; because the members of the supply chain want to improve their performance without considering how their decisions affect the other members of the supply chain. We refer to such supply chains as "decentralized supply chain". In a decentralized supply chain; there are at least two decision-makers, negatively impacting the performance of the supply chain. In recent years, supply chain members have been trying to coordinate themselves using contracts. So far, much work has been done to show that contracts can contribute to ensuring coordination in the supply chain [1,2]. Tsay and Nahmias [1] and Cachon [2] conducted two detailed studies on supply chain management and contracts. Tsay and Nahmias [1] discussed supply chain models, general objectives of contracts and their classification. According to Cachon [2], at the end of the sales season, the retailer makes a transfer payment to the supplier and the contracts are classified according to these transfer payments, such as repurchase contracts, revenue sharing contracts, quantity flexibility contracts, sales rebate contracts and quantity discount contracts.

Buyback contracts and revenue sharing contracts are the most studied supply chain contract types in the literature. In addition, there is a close relationship between the buyback contracts and the revenue sharing contracts. For these reasons; in this study we concentrated on the revenue sharing contract and buy back contract. Under the revenue sharing contract, the supplier offers a low selling price at the beginning of the selling season. At the end of the season; the retailer shares a part of the revenue from the sales with the supplier. Under the buyback contract, suppliers sell products to retailers at a wholesale price. At the end of the period, the products remaining in the possession of the retailer are returned to the supplier at a buyback price.

Various parameters such as demand, production costs, transfer costs, processing times, supply delivery, etc. may include uncertainty in a supply chain. In the management of a supply chain there are large number of uncertainty types that must be overcome, and it is even more difficult to coordinate the supply chain members under such circumtances. Traditional supply chain coordination models often emphasize the randomness of uncertainty. Several probabilistic models have been proposed to cope with this uncertainty in supply chain coordination. [3-7]. Rhee *et al.* [3] have stated that a revenue sharing contract must cover all members of the supply chain and proposed a comprehensive revenue sharing agreement for this purpose. Sarathi *et al.* [4]

have investigated a combined revenue sharing and quantity discount model. The demand in their model is price-sensitive and stock dependent. Kaya [5] analysed two different models for coordination; supplier effort (outsourcing) and manufacturer effort (in-house production) models. He investigated four contracts which are wholesale price, revenue sharing, quantity discount, cost sharing. Elahi *et al.* [6] have considered revenue sharing and buyback contracts between a rational supplier and a retailer. They have proposed five approaches to improve the retailer's decisions which are in response to contract terms offered by the supplier. Wu [7] has reviewed the impacts of the buyback contract on the retail price, the order quantity and the wholesale price in the supply chain. Chakraborty *et al.* [8] have considered a two-echelon supply chain where the final demand depends upon both the retail price and the marketing expenses borne by the partners. Hong and Guo [9] studied several cooperation contracts within a green product supply chain and investigate their environmental performance and analyzed and modeled under three contracts in order of increasing cooperation level: price-only, green-marketing cost-sharing, and two-part tariff contracts. The analytical results show that cooperation among partners can help the supply chain achieve environmental improvements. Guo *et al.* [10] aim to review in their paper the recent state of the literature on supply chain contracts with a focus on reverse logistics systems.

It is possible to model the probability distribution function in case of sufficient data. However, in the absence of sufficient data, it is not possible to construct a probability distribution function. Fuzzy set theory is an important solution when probability distributions cannot be generated for reasons such as lack of data. The methods based on the probability theory allow only quantitative uncertainties, in reality, most of the evaluations are imprecise and fuzzy and they cannot be quantified [11]. In recent years, fuzzy set theory has been frequently used in studies of supply chain contracts. Nevertheless, the number of studies involving situations in which demand quantities are expressed in fuzzy numbers in supply chain contracts is rather limited. Xu and Zhai [12] have investigated two-stage supply chain contracts with fuzzy demand comparing the centralized and decentralized supply chain and demonstrating that the expected maximum supply chain performance in the case of coordination is above the total performance in the uncoordinated situation. Zhang *et al.* [13] have investigated buyback contract under fuzzy random variable demand. In decentralized and centralized systems, optimal order quantities were estimated under the terms of a buyback contract. Taleizadeh *et al.* [14] have examined a newsvendor problem which covers multi-constraint, multi-product and fuzzy demand with quantity discount contract. Saha *et al.* [15] have formulated a revenue sharing contract in a fuzzy stochastic environment and multi-product multi-purpose supply chain models with limited risk and budget for the long term. The supply chain used

References	Supply Chain Problem				Contract Type
	Topology	Period	Decision Variable	Fuzzy Variable	
J. Zhao and J. Wei (2014) [34]	1-M/1-R	1-p	quantity variable,wholesale price	fuzzy demand	
A. Saha, S. Kar and M. Maiti (2015) [15]	1-M/1-R	n-p	quantity variable,wholesale price	fuzzy production cost,stochastic demand	revenue sharing contract
R. Xu and X. Zhai (2010) [12]	1-M/1-R	1-p	quantity variable	fuzzy demand	wholesale price contract
B. Zhang, S. Lu, D. Zhang and K. Wen (2014) [13]	1-M/1-R	1-p	quantity variable,wholesale price,buy back price	fuzzy demand	buy back contract
X. Ji and Z. Shao (2006) [35]	1-M/n-R	1-p	quantity variable	fuzzy demand	quantity discount contract
S. Sang (2014) [16]	1-M/1-D/1-R	1-p	quantity variable,wholesale price	fuzzy demand	spanning revenue sharing contract
K. Ryu and E. Yücesan (2010) [36]	1-M/1-R	1-p	quantity variable	fuzzy wholesale price,fuzzy demand,fuzzy retailer price	quantity discount contract, revenue sharing contract and buy back contract

TABLE 1
Supply chain contract models under fuzzy environment

in their study consists of a single manufacturer and a single retailer. Sang [16] has investigated return contracts and revenue sharing contracts with a supplier and multiple retailers in the event of fuzzy demand.. Arshinder *et al.* [17] have reviewed the applicability and paybacks of contracts and proposed various contract instruments. They then evaluated the success of these contracts. Studies on supply chain contracts under fuzzy environment are shown in Table 1.

It can be seen that; the work on coordination between supply chain members in a fuzzy environment is more limited than those in the stochastic environment. Therefore, this study, which will deal with supply chain contracts in cases where variables are expressed in fuzzy numbers, may contribute to the literature. In this study; a two-stage supply chain coordination based on revenue sharing contract and buy back contract under fuzzy environment is examined. Firstly, the effects of the revenue sharing contract and the buyback contract on supply chain performance are analyzed. The generalized closed form solutions of performance for these contracts were obtained using the expected value function defined by Xue *et al.* [18], and the credibility theory suggested by Liu and Liu [19] was applied. Then, we focused on the fuzziness aspect of the demand uncertainty.. Sensitivity analyses of these contracts are conducted by changing the contract parameters (revenue sharing rate ($\emptyset$) and buyback price (k)). Finally, numerical examples are presented to show the validity of the proposed method and to investigate the effects of the contract parameters on the total performance of the supply chain.

The rest of the paper is organized as follows. In the second part, definitions related to fuzzy set theory and credibility theory are mentioned. In section 3; fuzzy supply chain models with revenue sharing contract and buy back contract developed by using credibility theory are illustrated. Section 4 provides numerical examples to analyse the effects of the suggested models. And section 5 summarizes the study and shows its possible extensions.

2 THEORETICAL BACKGROUND

In the supply chain; there are many members, such as suppliers, retailers, distributors, etc. These members generally act as independent members and they must provide coordination among each other to increase the total supply chain performance. In recent years, the supply chain members want to manage this coordination by using contracts. In addition, there are many types of uncertainty that must be overcome in the management of the supply chain, and it is even more difficult to coordinate supply chain members under uncertainty. Therefore, this study examines the effects of the revenue sharing contract and the buyback contract on a two-stage supply chain model with fuzzy demand. Then the fuzziness of demand uncertainty is examined for each contract.

The expected value of a function of a fuzzy variable for a continuous membership function is used to determine the optimal order quantity that maximizes the total fuzzy performance of the supply chain. In this section, fuzzy set theory and credibility theory are explained for better understanding the proposed model in our work. Fuzzy sets are an extraordinary example of classical (net) sets put forward by Zadeh [20] as a mathematical tool to show uncertainty. And fuzzy sets are defined by membership functions. While in a classical set, the element of space belongs to or does not belong to a set; in fuzzy sets the degree of membership of each element changes in an interval. A fuzzy set can be mathematically expressed as $\widetilde{A} = \left(x, \mu_{\widetilde{A}}(x)\right), \forall x \in X$ where X is the universal set and $\mu_{\widetilde{A}}(x)$ is the membership function.

2.1 Fuzzy Variables and Credibility Theory

2.1.1 Fuzzy Variable

Fuzzy set theory has been used by researchers in more than one scientific field. Researchers have identified a fuzzy event using a fuzzy number [19]. A fuzzy number is a special case of a convex, normalized fuzzy set of the real line [21].

Let $\widetilde{X}$ be a generalized fuzzy number whose membership function is $\mu_{\widetilde{X}}(x)$. The membership function fulfils the following conditions with $0 < w \leq 1$ and $-\infty < l < m < n < u < \infty$ [22];

1. $\mu_{\widetilde{X}}(x)$ is a continuous mapping from (R), to the closed interval $[0, 1]$
2. $\mu_{\widetilde{X}}(x) = 0, -\infty < x < l$
3. $\mu_{\widetilde{X}}(x) = L(x)$, is strictly increasing on $[l, m]$
4. $\mu_{\widetilde{X}}(x) = w, m < x < n,$
5. $\mu_{\widetilde{X}}(x) = R(x)$,is strictly decreasing on $[n, u]$
6. $\mu_{\widetilde{X}}(x) = 0, u < x < u$

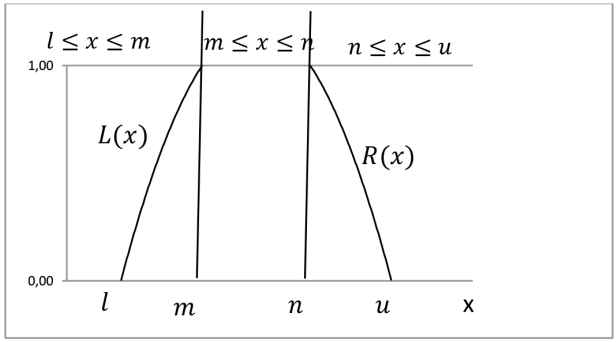

FIGURE 1
Membership function of a generalized fuzzy number

This type of generalized fuzzy number is denoted as $\widetilde{X} = (l; m; n; u; w)_{LR}$. When $w = 1$; it can be denoted as $\widetilde{X} = (l; m; n; u)_{LR}$.

The membership function for the generalized L-R type fuzzy number taking the above properties into account can be expressed by (1) and Figure 1.;

$$\mu_{\widetilde{X}}(x) = \begin{cases} L(x) & l \le x \le m \\ 1 & m \le x \le n \\ R(x) & n \le x \le u \\ 0 & \text{other} \end{cases} \tag{1}$$

2.1.2 Credibility Theory

The theory of possibility was introduced by Zadeh [21] in 1978 for mathematical expression of fuzziness. It has continuously been improved by many researchers such as Dubois and Prade [23]. Although having been improved by many researchers, it has no self-duality property. However, the property of self-duality is exactly needed in both practice and theory. In order to define this property, the concept of credibility theory has been improved.

In the possibility theory, there are two measures defining the uncertainty such as possibility and necessity measures. Possibility measure is thought as a parallel concept of probability measure. However, these two measures have partial differences. Necessity measure is the dual of possibility measure [23]. Nevertheless, there is no self-duality feature of neither possibility nor necessity measure. A measure with self-dual capability is needed both in practice and in theory. Credibility measure was introduced to define this self-dual feature by Liu and Liu in 2002 [19]. Therefore, the credibility measure shows characteristics similar to those of probability measure.

Credibility theory has been investigated by many researchers in many areas such as fuzzy scheduling problems, engineering design, programming algorithms, web mining, and expected value problems. Liu [24] proposed two programming algorithms, such as chance-constrained programming and dependent-chance programming, using reliability theory. Peng and Liu [25] have investigated parallel machine scheduling problem involving fuzzy processing times. Ji *et al.* [26] solved a shortest path problem in which the arc lengths are fuzzy variables. Yang and Sun [27] examined a warehouse layout problem by using expected value model in which some of the parameters are fuzzy random variable. Chen *et al.* [28] investigated a model by using fuzzy expected value model for determining target values of engineering characteristics.

Let ξ be a fuzzy variable with the membership function $\mu(x)$ and r be a real number,then the possibility and necessity measure of the fuzzy event $\{\xi \leq r\}$ can be respectively represented as in (2) and (3);

$$Pos\{\xi \leq r\} = supp_{x \leq r}\mu(x) \tag{2}$$

$$Nec\{\xi \leq r\} = 1 - Pos\{\xi \leq r\} = 1 - supp_{x \leq r}\mu(x) \tag{3}$$

The credibility measure which is introduced by Liu and Liu [19] is the average of possibility measure and necessity measure. The credibility measure of the fuzzy event $\{\xi \leq r\}$ can be represented as in (4);

$$Cr\{\xi \leq r\} = \frac{1}{2}(Pos\{\xi \leq r\} + Nec\{\xi \leq r\}) \tag{4}$$

Similarly, the credibility measure of the fuzzy event $\{\xi \geq r\}$ can be represented as in (5);

$$Cr\{\xi \geq r\} = \frac{1}{2}(Pos\{\xi \geq r\} + Nec\{\xi \geq r\}) \tag{5}$$

2.1.3 Expected Value of a Function of a Fuzzy Variable

The expected value of a fuzzy number is defined as the center of the expected interval [29]. Moreover, most of the ranking methods used for fuzzy numbers are used to find the expected value of a fuzzy number [30, 31, 32].

It has been a rather complicated process for researchers to analytically obtain the expected value of a function of a fuzzy number in the literature up to now [33]. Nevertheless, Xue *et al.* have developed an analytical method that uses the properties of the credibility theory to calculate the expected value of a function of a fuzzy variable [18].

Let ξ be a fuzzy variable with the membership function $\mu(x)$ and r be a real number, the expected value of a fuzzy variable $E[\xi]$ can be calculated as in (6) [19]:

$$E[\xi] = \int_0^\infty Cr\{\xi \geq r\}\, dr - \int_{-\infty}^0 Cr\{\xi \leq r\}\, dr \tag{6}$$

Let ξ be a fuzzy variable with the membership function $\mu_\xi(x)$, and $f:\Re \to \Re$ is a strictly monotonic function. If the Lebesque integrals, $\int_0^\infty Cr\{\xi \geq r\}\, dr$ and $\int_{-\infty}^0 Cr\{\xi \leq r\}\, dr$ are finite, then the expected value of a function of a fuzzy variable $E[f(\xi)]$ can be calculated as in (7) [18];

$$E[f(\xi)] = \int_{-\infty}^\infty f(r)\, dCr\{\xi \leq r\} \tag{7}$$

Let ξ be a fuzzy variable whose support is $[a, b]$ and $f:\Re \to \Re$ is a strictly monotonic function. If the Lebesque integrals, $\int_0^\infty Cr\{\xi \geq r\}\, dr$ and $\int_{-\infty}^0 Cr\{\xi \leq r\}\, dr$ are finite, then the expected value of a function of a fuzzy variable $E[f(\xi)]$ can be calculated as in (8) [18];

$$E[f(\xi)] = \int_a^b f(r)\, dCr\{\xi \leq r\} \tag{8}$$

3 PROBLEM FORMULATION

Let's consider a centralized supply chain model that consists of a retailer selling the product to the end customer and a supplier selling the product to the retailer. In this supply chain structure, the demand parameter cannot be obtained owing to insufficient historical data. Therefore, the demand parameter is regarded as a fuzzy variable $\left(\widetilde{X} = (l; m; n; u)\right)$. Consider a centralized supply chain structure, which can also be seen as a joint decision between the retailer and the supplier. In this situation, there is a decision-maker in the supply chain and the total performance of the supply chain can be maximized. Consequently, the supply chain members make a decision which deals not only with themselves but also with the entire supply chain.

In this supply chain; the supplier first offers a wholesale price to the retailer (w), the retailer sets the order quantity (Q) based on this wholesale price before the start of the sales season. And during the selling season; the

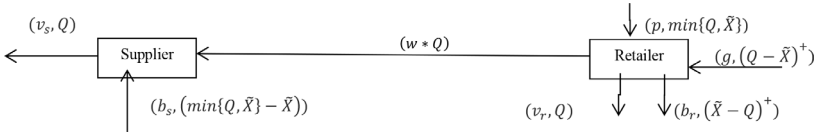

FIGURE 2
Structure of a centralized supply chain model

retailer sells the product to the end customer with a retail price(p). The supplier bears a production cost per unit such as v_s , and the retailer's marginal cost per unit is v_r where $v_s + v_r < p$. If the retailer's order quantity is less than demand; goodwill penalty cost for retailer (b_r) occurs, and the same cost for the supplier is b_s. For notational convenience, let $v = v_s + v_r \wedge b = b_r + b_s$. At the end of the sales season, the retailer earns a salvage value (g) for units that are not sold. In addition, at the end of the sales season, the retailer makes a transfer payment to the supplier. In a centralized supply chain structure, the transfer payment($w * Q$) depends on the wholesale price and the amount of the order quantity that the retailer has made at the start of the selling season. The representation of the structure of a centralized supply chain is shown in figure 2.

Because the demand parameter is a generalized fuzzy variable; the profit functions of the supplier and retailer have to be fuzzy functions. Let demonstrate the fuzzy profit function of the retailer and the fuzzy profit function of the supplier respectively such as $\widetilde{\Pi}_r(Q)$ and $\widetilde{\Pi}_s(Q)$.

The profit functions of these supply chain members can be obtained as;

$$\widetilde{\Pi}_r(Q) = p * min\{Q, X\} + g * (Q - X)^+ - b_r * (X - Q)^+ - v_r * Q - w * Q \tag{9}$$

$$\widetilde{\Pi}_s(Q) = b_s * \left(min\left\{Q, \widetilde{X}\right\} - \widetilde{X}\right) - v_s * Q + w * Q \tag{10}$$

After that; we obtained these functions as equation (11) and equation (12)

$$\widetilde{\Pi}_r(Q) = \begin{cases} p * \widetilde{X} + g * \left(Q - \widetilde{X}\right) - v_r * Q - w * Q & \widetilde{X} \le Q \\ p * Q - b_r * \left(\widetilde{X} - Q\right) - v_r * Q - w * Q & \widetilde{X} \ge Q \end{cases} \tag{11}$$

$$\widetilde{\Pi}_s(Q) = \begin{cases} -v_s * Q + w * Q & \widetilde{X} \le Q \\ b_s * \left(Q - \widetilde{X}\right) - v_s * Q + w * Q & \widetilde{X} \ge Q \end{cases} \tag{12}$$

The total profit of the supply chain is obtained the sum of the profits of the members in the supply chain. In our study; the supply chain consists of two

members; such as a retailer and a supplier. So the total profit of this supply chain equals the sum of the profits of the retailer and supplier, and the total profit of the supply chain can be obtained as follows:

$$\widetilde{\Pi}(Q) = \widetilde{\Pi}_r(Q) + \widetilde{\Pi}_s(Q)$$

$$\widetilde{\Pi} = \begin{cases} p * \widetilde{X} + g * \left(Q - \widetilde{X}\right) - Q * v & \widetilde{X} \leq Q \\ p * Q - b * \left(\widetilde{X} - Q\right) - Q * v & \widetilde{X} \geq Q \end{cases} \tag{13}$$

In the centralized supply chain structure; there is a decision maker wants to optimize the performance of whole supply chain. This decision maker only wants to reduce the costs and increase the profits of the supply chain members. So, the decision maker must obtain the optimum order quantity which maximizes the total supply chain profit function. In order to find the optimal order quantity value that maximizes the total supply chain profit function, it is necessary to take the derivative of the total supply chain profit function according to the order quantity variable. Optimum order quantity value (Q^*) is the order quantity value that equal to the zero obtained as a result of this derivative operation.

The optimum order quantity and the expected total supply chain profit in fuzzy centralized supply chain model is obtained as equation (A12) and equation (A13) in appendix A;

$$Q^* = \begin{cases} L^{-1}\left(\dfrac{2(p-v+b)}{p-g+b}\right); & p-v+b \leq v-g \\ [m,n]; & p-v+b = v-g \\ R^{-1}\left(\dfrac{2(v-g)}{p-g+b}\right); & p-v+b \geq v-g \end{cases} \tag{14}$$

$$E\left[\widetilde{\Pi}\left(Q^*\right)\right] =$$

$$\begin{cases} (p-g)\int_l^Q rdCr\left\{\widetilde{X} \leq r\right\} - b\int_Q^m rdCr\left\{\widetilde{X} \leq r\right\} & \\ \quad -b\int_n^u rdCr\left\{\widetilde{X} \leq r\right\} & ; p-v+b \leq v-g \\ p\int_l^m rdCr\left\{\widetilde{X} \leq r\right\} - g\int_l^m rdCr\left\{\widetilde{X} \leq r\right\} & \\ \quad -b\int_n^u rdCr\left\{\widetilde{X} \leq r\right\} & ; p-v+b = v-g \\ (p-g)\int_l^m rdCr\left\{\widetilde{X} \leq r\right\} + (p-g)\int_n^Q rdCr\left\{\widetilde{X} \leq r\right\} & \\ \quad -b\int_Q^u rdCr\left\{\widetilde{X} \leq r\right\} & ; p-v+b \geq v-g \end{cases} \tag{15}$$

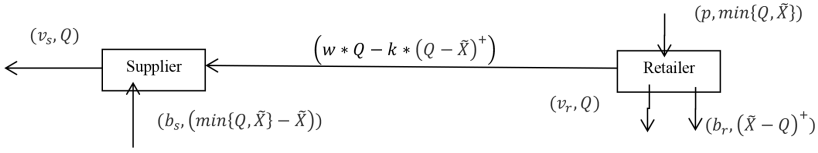

FIGURE 3
Structure of a supply chain model under buyback contract

In the previous supply chain structure; supply chain members act as a single decision maker and want to optimize the performance of the entire supply chain. But sometimes this cannot happen because the members of the supply chain want to improve their performance without considering how their decisions affect the other members of the supply chain. We call this supply chain as a "decentralized supply chain". In a decentralized supply chain; there are at least two members who make decisions and this situation affects the performance of supply chain negatively. In such a situation, the supply chain members want to manage the coordination between each other by using contracts. As mentioned before; in this study we investigate two supply chain contracts; such as buyback contract and revenue sharing contract. According to these contracts, at the end of the selling season, a transfer payment is made to the supplier form the retailer and the contracts differ according to these transfer payments.

Under the buyback contract, the retailer is responsible for the wholesale price for each product that the retailer buys from the supplier through the buyback contract. At the end of the sales season, the supplier repays a lower buyback price ($k \leq w$) for each product that the retailer can not sell at a wholesale price. The retailer should not earn from the remaining units that it cannot sell. The transfer payment for the buyback contract is based on the buyback price paid by the supplier for the remaining units at the end of the season. According to this contract; the salvage value of unsold products does not occur because all products that are not sold are taken back by the supplier. The representation of the structure of a supply chain under buy back contract is shown in figure 3.

The profit functions of these supply chain members can be obtained as;

$$\begin{aligned} \widetilde{\Pi}_r(Q) &= p * min\{Q, X\} - b_r * (X - Q)^+ - v_r * Q \\ &\quad - \left(w * Q - k * \left(Q - \widetilde{X}\right)^+\right) \end{aligned} \tag{16}$$

$$\widetilde{\Pi}_s(Q) = b_s * \left(min\left\{Q, \widetilde{X}\right\} - \widetilde{X}\right) - v_s * Q + \left(w * Q - k * \left(Q - \widetilde{X}\right)^+\right) \tag{17}$$

After that; we obtained these functions as equation (18) and equation (19)

$$\widetilde{\Pi}_r(Q) = \begin{cases} p * \widetilde{X} - v_r * Q - w * Q + k * \left(Q - \widetilde{X}\right) & \widetilde{X} \le Q \\ p * Q - b_r * \left(\widetilde{X} - Q\right) - v_r * Q - w * Q & \widetilde{X} \ge Q \end{cases} \tag{18}$$

$$\widetilde{\Pi}_s(Q) = \begin{cases} -v_s * Q + w * Q - k * \left(Q - \widetilde{X}\right) & \widetilde{X} \le Q \\ b_s * \left(Q - \widetilde{X}\right) - v_s * Q + w * Q & \widetilde{X} \ge Q \end{cases} \tag{19}$$

In the decentralized supply chain structure; there are at least two members who can make decisions. But in such a structure; the retailer is fully responsible for the risk of inconsistency between demand $\left(\widetilde{X}\right)$- order quantity (Q). So the decision maker must obtain the optimum order quantity which maximizes the retailer profit function. In order to find the optimal order quantity value that maximizes the retailer profit function, it is necessary to obtain the derivative of the profit function of the retailer according to the order quantity variable. Optimum order quantity value (Q^*) is the order quantity value that equal to the zero obtained as a result of this derivative operation.

The optimum order quantity and the profits of the retailer and supplier in fuzzy supply chain model under the buyback contract is obtained as equation (B15), equation (B16) and equation (B17) in appendix B;

$$Q^* = \begin{cases} L^{-1}\left(\dfrac{2(w + v_r - p - b_r)}{k - p - b_r}\right); & 2w + 2v_r - p \le k + b_r \\ [m, n]; & 2w + 2v_r - p = k + b_r \\ R^{-1}\left(\frac{2(k - w - v_r)}{k - p - b_r}\right); & 2w + 2v_r - p \ge k + b_r \end{cases} \tag{20}$$

$$E\left[\widetilde{\Pi}_r\left(Q^*\right)\right] = \begin{cases} (p-k)\int_l^Q r dCr\left\{\widetilde{X} \le r\right\} & \\ \quad -b_r\left[\int_Q^m r dCr\left\{\widetilde{X} \le r\right\} + \int_n^u r dCr\left\{\widetilde{X} \le r\right\}\right] & 2w + 2v_r - p \le k + b_r \\ (p-k)\int_l^m r dCr\left\{\widetilde{X} \le r\right\} - b_r \int_n^u r dCr\left\{\widetilde{X} \le r\right\} & 2w + 2v_r - p = k + b_r \\ (p-k)\left[\int_l^m r dCr\left\{\widetilde{X} \le r\right\} + \int_n^Q r dCr\left\{\widetilde{X} \le r\right\}\right] & \\ \quad -b_r \int_Q^u r dCr\left\{\widetilde{X} \le r\right\} & 2w + 2v_r - p \ge k + b_r \end{cases} \tag{21}$$

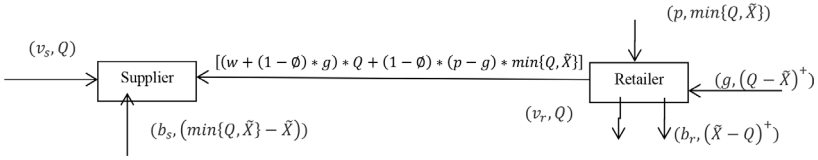

FIGURE 4
Structure of a supply chain model under revenue sharing contract

$$E\left[\tilde{\Pi}_s(Q^*)\right] =$$

$$\begin{cases} k\int_l^Q rdCr\left\{\tilde{X} \le r\right\} - b_s\left[\int_Q^m rdCr\left\{\tilde{X} \le r\right\} + \int_n^u rdCr\left\{\tilde{X} \le r\right\}\right] & 2w + 2v_r - p \le k + b_r \\ + \left[(-k - b_s)\left(\dfrac{w + v_r - p - b_r}{k - p - b_r}\right) + (b_s - v_s + w)\right] Q; & \\ k\int_l^m rdCr\left\{\tilde{X} \le r\right\} - b_s\int_n^u rdCr\left\{\tilde{X} \le r\right\} & 2w + 2v_r - p = k + b_r \\ + \left[(-k - b_s)\left(\dfrac{w + v_r - p - b_r}{k - p - b_r}\right) + (b_s - v_s + w)\right] Q; & \\ k\left[\int_l^m rdCr\left\{\tilde{X} \le r\right\} + \int_n^Q rdCr\left\{\tilde{X} \le r\right\}\right] - b_s\int_Q^U rdCr\left\{\tilde{X} \le r\right\} & 2w + 2v_r - p \ge k + b_r \\ + \left[(-k - b_s)\left(\dfrac{w + v_r - p - b_r}{k - p - b_r}\right) + (b_s - v_s + w)\right] Q; & \end{cases} \tag{22}$$

Another contract for the coordination of the supply chain is the revenue sharing contract. Under this contract, the retailer pays the supplier a wholesale price per unit purchased. On the other hand, the retailer shares some of the revenue ($\emptyset$) at the end of the season with the supplier. The transfer payment for revenue sharing contract depends on the revenue sharing fraction ($\emptyset$). The representation of the structure of a supply chain under revenue sharing contract is shown in figure 4.

The profit functions of these supply chain members can be obtained as;

$$\tilde{\Pi}_r(Q) =$$

$$p * min\{Q, X\} - b_r * (X - Q)^+ + g * \left(Q - \tilde{X}\right)^+ - v_r * Q \tag{23}$$
$$- \left[(w + (1 - \emptyset) * g) * Q + (1 - \emptyset) * (p - g) * min\left\{Q, \tilde{X}\right\}\right]$$

$$\tilde{\Pi}_s(Q) =$$

$$b_s * \left(min\left\{Q, \tilde{X}\right\} - \tilde{X}\right) - v_s * Q + [(w + (1 - \emptyset) * g) * Q \tag{24}$$
$$+ (1 - \emptyset) * (p - g) * min\left\{Q, \tilde{X}\right\}]$$

After some calculations; we obtained these functions as equation (25) and equation (26)

$$\widetilde{\Pi}_r(Q) = \begin{cases} p * \emptyset * \widetilde{X} + g * \emptyset * \left(Q - \widetilde{X}\right) - v_r * Q - w * Q & \widetilde{X} \le Q \\ p * \emptyset * Q - b_r * \left(\widetilde{X} - Q\right) - v_r * Q - w * Q & \widetilde{X} \ge Q \end{cases} \tag{25}$$

$$\widetilde{\Pi}_s(Q) = \begin{cases} -v_s * Q + w * Q + (1-\emptyset) * \left(Q - \widetilde{X}\right) * g + (1-\emptyset) * p * \widetilde{X} & \widetilde{X} \le Q \\ b_s * \left(Q - \widetilde{X}\right) - v_s * Q + w * Q + (1-\emptyset) * p * Q & \widetilde{X} \ge Q \end{cases} \tag{26}$$

As mentioned in the buyback contract; in the decentralized supply chain structure; there are at least two members who can make decisions. But in such a structure; the retailer is fully responsible for the risk of inconsistency between demand $\left(\widetilde{X}\right)$- order quantity (Q) . Therefore, the decision maker must obtain the optimum order quantity which maximizes the retailer profit function. In order to find the optimal order quantity value that maximizes the retailer profit function, it is necessary to obtain the derivative of the profit function of the retailer according to the order quantity variable.. Optimum order quantity value (Q^*) is the order quantity value that equal to the zero obtained as a result of this derivative operation.

The optimum order quantity and the profits of the retailer and supplier in fuzzy supply chain model under the revenue sharing contract is obtained as equation (C15), equation (C16) and equation (C17) in appendix C;

$$Q^* = \begin{cases} L^{-1}\left(\frac{2*(w+v_r-p\emptyset-b_r)}{g*\emptyset-p*\emptyset-b_r}\right); & 2w + 2v_r - p\emptyset \ge g\emptyset + b_r \\ [m, n]; & 2w + 2v_r - p\emptyset \ge g\emptyset + b_r \\ R^{-1}\left(\frac{2(g\emptyset-w-v_r)}{g*\emptyset-p*\emptyset-b_r}\right); & 2w + 2v_r - p\emptyset \ge g\emptyset + b_r \end{cases} \tag{27}$$

$$E\left[\widetilde{\Pi}_r\left(Q^*\right)\right] = \begin{cases} (p-g)\emptyset \int_l^Q r dCr\left\{\widetilde{X} \le r\right\} - b_r \left[\int_Q^m r dCr\left\{\widetilde{X} \le r\right\} \right. \\ \quad \left. + \int_n^u r dCr\left\{\widetilde{X} \le r\right\}\right] & 2w + 2v_r - p\emptyset \ge g\emptyset + b_r \\ (p-g)\emptyset \int_l^m r dCr\left\{\widetilde{X} \le r\right\} - b_r \int_n^u r dCr\left\{\widetilde{X} \le r\right\} & 2w + 2v_r - p\emptyset \ge g\emptyset + b_r \\ (p-g)\emptyset \left[\int_l^m r dCr\left\{\widetilde{X} \le r\right\} + \int_n^Q r dCr\left\{\widetilde{X} \le r\right\}\right] \\ \quad -b_r \int_Q^u r dCr\left\{\widetilde{X} \le r\right\} & 2w + 2v_r - p\emptyset \ge g\emptyset + b_r \end{cases} \tag{28}$$

$$E\left[\widetilde{\Pi}_s\left(Q^*\right)\right] =$$

$$\begin{cases} (p-g)(1-\emptyset)\int_{l}^{Q} r dCr\left\{\widetilde{X}\le r\right\} & \\ -b_s\left[\int_{Q}^{m} r dCr\left\{\widetilde{X}\le r\right\}+\int_{n}^{u} r dCr\left\{\widetilde{X}\le r\right\}\right] & 2w+2v_r-p\le k+b_r \\ +\left[\left(-b_s-(p-g)(1-\emptyset)\right)\left(\frac{(w+v_r-p\emptyset-b_r)}{g*\emptyset-p*\emptyset-b_r}\right)\right. & \\ \left.+(b_s-v_s+w+(1-\emptyset)p)\right]Q; & \\ (p-g)(1-\emptyset)\int_{l}^{m} r dCr\left\{\widetilde{X}\le r\right\}-b_s\int_{n}^{u} r dCr\left\{\widetilde{X}\le r\right\} & 2w+2v_r-p=k+b_r \\ +\left[\left(-b_s-(p-g)(1-\emptyset)\right)\left(\frac{(w+v_r-p\emptyset-b_r)}{g*\emptyset-p*\emptyset-b_r}\right)\right. & \\ \left.+(b_s-v_s+w+(1-\emptyset)p)\right]Q; & \\ (p-g)(1-\emptyset)\left[\int_{l}^{m} r dCr\left\{\widetilde{X}\le r\right\}+\int_{n}^{Q} r dCr\left\{\widetilde{X}\le r\right\}\right] & 2w+2v_r-p\ge k+b_r \\ -b_s\int_{Q}^{U} r dCr\left\{\widetilde{X}\le r\right\}+\left[\left(-b_s-(p-g)(1-\emptyset)\right)\left(\frac{(w+v_r-p\emptyset-b_r)}{g*\emptyset-p*\emptyset-b_r}\right)\right. & \\ \left.+(b_s-v_s+w+(1-\emptyset)p)\right]Q; & \end{cases} \tag{29}$$

4 NUMERICAL EXAMPLES

In this section; numerical examples are given for the demonstration of the proposed models. Firstly, the effects of the revenue sharing contract and the buyback contract on supply chain performance are analysed. The profit values under two contracts and also the profit value in the centralized supply chain are compared. After that we focus on the fuzziness aspect of the demand uncertainty under each contract. In the other analyses; sensitivity analyses are conducted for proposed models by changing the contracts parameters (revenue sharing rate ($\emptyset$) and buyback price (k)).

In numerical analysis; we choose a product sold in a firm .The firm is an online retailer and it has one supplier for the selected product. The demands per month of this product are determined with trapezoidal fuzzy number as $\left(\widetilde{X}=95-\Delta,95,95+\beta,95+\alpha\right)$ with $\Delta\ge 0,\beta\ge 0$ and $\alpha\ge 0$ and also $\alpha\ge\beta$. Here as Δ, β and α increases; the fuzziness of the demand increases; furthermore as Δ, β and α decreases; the fuzziness of the demand decreases. In this supply chain; firstly the supplier proposes a wholesale price to the retailer ($w=50$ €) and the retailer decide the order quantity (Q) which maximizes the supply chain total profit. The retail price of this product, p, is 140 €.The salvage value of this product, g, is 10 €. And the other parameters used in this model are as shown in table 2.

Parameters		Value
v_r	The retailer's marginal cost per unit	30
v_s	The supplier's production cost per unit	10
b_r	Goodwill penalty cost for retailer	10
b_s	Goodwill penalty cost for supplier	20

TABLE 2
Contract parameter values

For notational convenience; we say that

$$v = v_r + v_s = 30 + 10 = 40 \text{ and } b = b_r + b_s = 10 + 20 = 30$$

ANALYSIS A:

In the centralized supply chain structure; there is a decision maker wants to optimize the performance of whole supply chain. This decision maker only wants to reduce the costs and increase the profits of the supply chain members. But this situation sometimes cannot be obtained because the members of the supply chain want to improve their performance without considering how their decisions affect other members of the supply chain. In such a situation, the supply chain members want to manage the coordination between each other by using contracts. In this section; the effects of the revenue sharing contract and the buyback contract on supply chain performance are analyzed. According to these contracts, at the end of the selling season, a transfer payment is made to the supplier form the retailer and the contracts differ according to these transfer payments.

It is assumed that the demand is a trapezoidal fuzzy variable with $(75, 95, 100, 115)$ with $\Delta = 20, \beta = 5, \alpha = 20$. And also, in buy back contract the supplier buys products that cannot be sold by retailer at the end of the season with buyback price $k = 40$. And at the revenue sharing contract the retailer pays a portion ($\emptyset = 0,8$ of the revenue to the supplier at the end of the season. The total profit of the supply chain is calculated with this demand variable as in equation (A13) in appendix A. And the profits of the retailer and supplier under buyback contract are calculated respectively as in equation (B16) and equation (B17) in appendix B; the profit of the retailer and the supplier is calculated according to the revenue sharing contract as if they are in equations (C16) and (C17) in appendix C respectively. The results are illustrated in Table 3.

While examining the supply chain model including one of the contracts and the centralized supply chain model without any contracts; the total profit of the supply chain in the centralized model seems to be higher than the total

	Order Quantity	Retailer Profit	Supplier Profit	Supply Chain Total Profit
Centralized Model	109			9.147
Buyback Contract	100	5.178	1.446	6.625
Revenue Sharing Contract	96	2.412	5.176	7.588

TABLE 3
Expected profits of retailer and supplier and optimum order quantity ($k = 40$ and $\emptyset = 0, 8$)

profit of the supply chain containing any contracts. Also, the most profitable member in the supply chain varies according to contracts While the retailer gets higher revenues from the supplier on the buyback contract; the supplier earns higher profits from the retailer on a revenue sharing contract. And it can be seen that; the order quantity in centralized model is higher than the order quantity in supply chain model with any contract.

ANALYSIS B:

One of the important points in supply chain management is to obtain the demand value. When there is not enough historical data; demand variant can be shown using fuzzy set theory.So in this section; we focus on the fuzziness aspect of the demand uncertainty. As a numerical example, we suppose that the fuzzy demand is a trapezoidal fuzzy variable $\widetilde{X} = (95 - \Delta, 95, 95 + \beta, 95 + \alpha)$ with $\Delta \geq 0, \beta \geq 0$ and $\alpha \geq 0$ and also $\alpha \geq \beta$. And the parameter values are the same as the parameter values used in the previous analysis. Retailer's profit, supplier's profit, supply chain total profit and optimum order quantity values obtained by changing the demand variance are shown in table 4 and table 5.

It is seen from the tables that in the case of diminishing demand uncertainty profit values are increased according to both contracts. According to

				UNDER BUYBACK CONTRACT			
l	**m**	**n**	**u**	**ORDER QUANTITY**	**RETAILER PROFIT**	**SUPPLIER PROFIT**	**SUPPLY CHAIN TOTAL PROFIT**
75	95	100	115	100	5.178	1.446	6.625
65	95	100	125	103	4.924	1.308	6.231
55	95	100	135	106	4.669	1.169	5.838
45	95	100	145	109	4.415	1.030	5.445
35	95	100	155	111	4.160	891	5.051

TABLE 4
Retailer's profit, supplier's profit, supply chain total profit and optimum order quantity values under buyback contract

				UNDER BUYBACK CONTRACT			
l	**m**	**n**	**u**	**ORDER QUANTITY**	**RETAILER PROFIT**	**SUPPLIER PROFIT**	**SUPPLY CHAIN TOTAL PROFIT**
75	95	100	115	96	2.412	5.176	7.588
65	95	100	125	93	2.147	4.912	7.059
55	95	100	135	91	1.882	4.648	6.529
45	95	100	145	88	1.616	4.383	6.000
35	95	100	155	86	1.351	4.119	5.470

TABLE 5
Retailer's profit, supplier's profit, supply chain total profit and optimum order quantity values under revenue sharing contract

the buyback contract; the total profit of the supply chain is decreasing despite the increase in the order quantity. This is because the amount of product that cannot be sold and which has to be taken by the supplier increases as the amount of the order increases. As a result of this situation the retailer's profit decreases.

Under the revenue sharing contract; because of the uncertainty; the retailer wants to order less quantity. As the order quantity decreases; the supplier's profit decreases according to this reduction because of the scale economy.

ANALYSIS C:

In this section, sensitivity analyses are conducted for illustrating the validity of the proposed models by changing the contracts parameters; revenue sharing rate ($\emptyset$) for revenue sharing contract; buyback price (k) for buyback contract. It is assumed that the demand is a trapezoidal fuzzy variable with $(75, 95, 100, 115)$. Retailer profits and supplier profits obtained by changing the buyback price on the buyback contract are shown in Table 6.

It is observed that as buyback price (k) increases constantly, the profits of the supplier and the retailer increase. So the retailer wants to order more quantity because of the higher buyback price. But as the order quantity decreases;

Buyback Price	**Order Quantity**	**Retailer Profit**	**Supplier Profit**	**Supply Chain Total Profit**
40	100	5.178	1.446	6.625
35	99	5.140	1.020	6.160
30	98	5.104	634	5.738
25	97	5.072	282	5.354

TABLE 6
Retailer's profit and supplier's profit by changing the buyback price under buyback contract

Revenue Sharing Rate	Order Quantity	Retailer Profit	Supplier Profit	Supply Chain Total Profit
0,8	96	2412	5176	7588
0,7	93	1155	5015	6170
0,6	90	-69	4078	4009
0,5	85	-1244	2063	819

TABLE 7
Retailer's profit and supplier's profit by changing the revenue sharing rate in revenue sharing contract

the total supply chain profit also decreases. Because the retailer and the supplier profit values decrease.

The other sensitivity analysis about this section is the relationship between the revenue sharing rate value and the profits of the supply chain members. Retailer and supplier profits obtained by changing the revenue sharing ratio Ø in the revenue sharing contract are shown in table 7.

It is observed that as revenue sharing rate increases, the difference between the supplier and retailer profits decrease. Therefore, the higher the revenue sharing rate, the more compatible the supply chain members.

CONCLUSIONS

Supply chain contracts deal with finding the optimum supply chain performance by investigating the transfer payment under stochastic demand. However, in real life, sometimes probability distribution of the demand for products is difficult to obtain due to the lack of information. This study focuses on the possibility situations, where the demand is described by a membership function.

This article presents an analytical method to obtain the fuzzy expected value of supply chain total profit and the profits of the retailer and the supplier. To determine the optimum order quantity that maximizes the fuzzy total profit function, the expected value of a fuzzy function based on the credibility theory is employed. In addition, closed form solutions have been obtained for fuzzy profit functions and order quantities of buyback and revenue sharing contracts. Generalized closed-form solutions provide some advantages such as; they reduce the need for enumeration over alternative values and give the opportunity to analyse the effects of model parameters on optimum order quantity and optimum profit.

According to results of numerical analysis; while examining the supply chain model including one of the contracts and the centralized supply chain

model without any contracts; the total profit of the supply chain in the centralized model seems to be higher than the total profit of the supply chain containing any contracts. Also, the most profitable member in the supply chain varies according to contracts. While the retailer gets higher revenues from the supplier on the buyback contract; the supplier earns higher profits from the retailer on a revenue sharing contract. And it can be seen that the order quantity in centralized model is higher than the order quantity in supply chain model with any contract.

And with numerical analysis in the case of diminishing demand uncertainty, it is seen that profit values are increased according to both contracts. According to the buyback contract; the total profit of the supply chain is decreasing despite the increase in the order quantity. The reason for this is that the amount of product that cannot be sold and which has to be taken by the supplier increases as the amount of the order increases. Consequently, the retailer's profit decreases. Under the revenue sharing contract; because of the uncertainty; the retailer wants to order less quantity. As the order quantity decreases, the supplier's profit decreases according to this reduction because of the scale economy.

And according to under numerical analysis, it is observed that as buyback price (k) increases constantly, the profits of the supplier and the retailer increase. So the retailer wants to order more quantity because of the higher buyback price. But as the order quantity decreases; the total supply chain profit also decreases. Because the retailer and the supplier profit values decrease. It is observed that as revenue sharing rate increases; the difference between the supplier and retailer profits decrease. Therefore, the higher the revenue sharing rate, the more compatible the supply chain members.

The proposed models analysed in this paper consider two types of contracts which are buyback contract and revenue sharing contract. The models can be extended for the other type of contracts such as sales rebate contract, quantity discount contract, etc. Another suggestion for the future studies is that; future models can be extended including more than one product, multiple-period, more than one retailer and more than one supplier; while these models analysed for a two-stage supply chain including single-product; single-period, one retailer and one supplier.

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APPENDICES

APPENDIX A- CENTRALIZED FUZZY SUPPLY CHAIN MODEL

Let suppose that the demand $\widetilde{X}$ is a generalized L-R type fuzzy variable with membership function $\mu_{\widetilde{X}}(x)$ as in equation (1). Here $[m, n]$ are the most likely values of fuzzy number $\widetilde{X}$; l and u are the lower and upper values respectively. Since the fuzzy demand $\widetilde{X} = (l; m; n; u)$ is a positive generalized L-R type fuzzy number, we know that the order quantity Q has three cases; that is, $Q \in [l, m]$, $Q \in [m, n]$ or $Q \in [n, u]$

By using equation (7) and equation (8); the expected value of the total profit function $E\left[\widetilde{\Pi}(Q)\right]$ of fuzzy demand $\widetilde{X}$ will be formulated as in (A1);

$$
\begin{aligned}
E\left[\widetilde{\Pi}(Q)\right] &= \int_{l}^{Q} [p * r + g * (Q - r) - Q * v] dCr\left\{\widetilde{X} \leq r\right\} \\
&\quad + \int_{Q}^{u} [p * Q - b * (r - Q) - Q * v] dCr\left\{\widetilde{X} \leq r\right\} \\
&= -Q * v + p \int_{l}^{Q} r dCr\left\{\widetilde{X} \leq r\right\} + gQ \int_{l}^{Q} dCr\left\{\widetilde{X} \leq r\right\} \\
&\quad - g \int_{l}^{Q} r dCr\left\{\widetilde{X} \leq r\right\} + pQ \int_{Q}^{u} dCr\left\{\widetilde{X} \leq r\right\} - b \int_{Q}^{u} r dCr\left\{\widetilde{X} \leq r\right\} \\
&\quad + bQ \int_{Q}^{u} dCr\left\{\widetilde{X} \leq r\right\} \qquad \text{(A1)}
\end{aligned}
$$

The optimal order quantity Q^* will be the value that maximizes the expected value of fuzzy total profit function. To find the maximizing value of Q, we set;

$$\frac{dE\left[\widetilde{\Pi}(Q)\right]}{dQ} = 0$$

Thus, the following equations are obtained,

$$
\begin{aligned}
&\frac{dE\left[\widetilde{\Pi}(Q)\right]}{dQ} = -v + g \int_{l}^{Q} dCr\left\{\widetilde{X} \leq r\right\} + p \int_{Q}^{u} dCr\left\{\widetilde{X} \leq r\right\} + b \int_{Q}^{u} dCr\left\{\widetilde{X} \leq r\right\} \\
&0 = -v + (g - p - b) * Cr\left\{\widetilde{X} \leq Q\right\} + p + b \\
&Cr\left\{\widetilde{X} \leq Q\right\} = \frac{p - v + b}{p - g + b} \qquad \text{(A2)}
\end{aligned}
$$

To find the optimum order quantity, we will determine the value of $Cr\left\{\widetilde{X} \leq Q\right\}$. The credibility of fuzzy demand with the membership function as in (1) will be as in (A3);

$$Cr\left\{\widetilde{X} \leq Q\right\} = \begin{cases} 0 & Q \leq l \\ \dfrac{L(Q)}{2} & l \leq Q \leq m \\ \dfrac{1}{2} & m \leq Q \leq n \\ 1 - \dfrac{R(Q)}{2} & n \leq Q \leq u \\ 1 & Q \geq u \end{cases} \tag{A3}$$

There are three cases to be analyzed for the value of Q in discussing the credibility value of fuzzy demand;

Case 1:$l \leq Q \leq m$

$$Cr\left\{\widetilde{X} \leq Q\right\} = \frac{p - v + b}{p - g + b} = \frac{L(Q)}{2} \tag{A4}$$

Thus, the following equations are obtained;

$$L(Q) = \frac{2(p - v + b)}{p - g + b} \text{ and } Q^{=} L^{-1}\left(\frac{2(p - v + b)}{p - g + b}\right) \tag{A5}$$

Here, the value of $\frac{2(p-v+b)}{p-g+b}$ must be between $[0, 1]$, thus we can say that under the conditions of $p + b \geq v$ and $p - v + b \leq v - g$, Q^* will be in $[l, m]$. Moreover, the second derivative of $E\left[\widetilde{\Pi}(Q^*)\right]$ with respect to Q is;

$$\frac{d^2 E\left[\widetilde{\Pi}(Q^*)\right]}{dQ} = \frac{g - p - b}{2} L^{'}(Q)$$

Since $L(Q)$is an increasing function in $[l, m]$,we find that $L^{'}(Q) > 0$. The values of g, p and b are positive and $g < p + b$, so that we obtain $\frac{d^2 E\left[\widetilde{\Pi}(Q^*)\right]}{dQ} < 0$ which implies that Q^* is the optimum value which maximizes $E\left[\widetilde{\Pi}(Q^*)\right]$. The expected fuzzy total profit value for the optimum

order quantity will be as below;

$$E\left[\widetilde{\Pi}\left(Q^{*}\right)\right]=(p-g)\int_{l}^{L^{-1}\left(\frac{2(p-v+b)}{p-g+b}\right)} rd\left(\frac{L\left(r\right)}{2}\right)-b\int_{L^{-1}\left(\frac{2(p-v+b)}{p-g+b}\right)}^{m} rd\left(\frac{L\left(r\right)}{2}\right)$$

$$-\quad b\int_{n}^{u} rd\left(1-\frac{R\left(r\right)}{2}\right) \tag{A6}$$

Case 2: $m \leq Q \leq n$

$$Cr\left\{X \leq Q\right\}=\frac{p-v+b}{p-g+b}=\frac{1}{2} \tag{A7}$$

If there exists $p-v+b=v-g$ case, then $E\left[\widetilde{\Pi}\left(Q\right)\right]$ will be maximized by any $Q \in [m, n]$.Thereby, under the condition of $p-v+b=v-g$, we can say that $Q^{*} \in [m, n]$.The expected fuzzy total profit value for the optimum order quantity will be as in (A8);

$$E\left[\widetilde{\Pi}\left(Q^{*}\right)\right]=p\int_{l}^{m} rd\left(\frac{L\left(r\right)}{2}\right)-g\int_{l}^{m} rd\left(\frac{L\left(r\right)}{2}\right)-b\int_{n}^{u} rd\left(1-\frac{R\left(r\right)}{2}\right) \tag{A8}$$

Case 3: $n \leq Q \leq u$

$$Cr\left\{\widetilde{X} \leq Q\right\}=\frac{p-v+b}{p-g+b}=1-\frac{R\left(Q\right)}{2} \tag{A9}$$

And the following equations are obtained,

$$R\left(Q\right)=\frac{2\left(v-g\right)}{p-g+b} \text{ and } Q^{*}=R^{-1}\left(\frac{2\left(v-g\right)}{p-g+b}\right) \tag{A10}$$

Here, the value of $\frac{2(v-g)}{p-g+b}$ must be between [0, 1], thus we can say that under the condition of $p-v+b \geq v-g$, Q^{*} will be in $[n, u]$. Additionally, the second derivative of $E\left[\widetilde{\Pi}\left(Q^{*}\right)\right]$ with respect to Q is;

$$\frac{d^{2}E\left[\widetilde{\Pi}\left(Q^{*}\right)\right]}{dQ}=\frac{-g-p-b}{2}R^{'}\left(Q\right)$$

Since $R(Q)$ is an decreasing function in $[n,u]$, we find that $R^{'}(Q)<0$. The values of g, p and b are positive and $g<p+b$, so that we obtain $\frac{d^2E\left[\widetilde{\Pi}(Q^*)\right]}{dQ}<0$ which implies that Q^* is the optimum value which maximizes $E\left[\widetilde{\Pi}(Q^*)\right]$

The expected fuzzy total profit value for the optimum order quantity will be as below;

$$E\left[\widetilde{\Pi}\left(Q^*\right)\right]=(p-g)\int_l^m rd\left(\frac{L(r)}{2}\right)+(p-g)\int_n^{R^{-1}\left(\frac{2(v-g)}{p-g+b}\right)} rd\left(1-\frac{R(r)}{2}\right)$$

$$-b\int_{R^{-1}\left(\frac{2(v-g)}{p-g+b}\right)}^{u} rd\left(1-\frac{R(r)}{2}\right) \tag{A11}$$

If the demand is a generalized L-R type fuzzy number with the membership function in (1), the generalized closed form solutions of the optimum order quantity and the profit of the total supply chain in centralized decision model is calculated as;

$$Q^*=\begin{cases} L^{-1}\left(\frac{2(p-v+b)}{p-g+b}\right); & p-v+b\le v-g \\ [m,n]; & p-v+b=v-g \\ R^{-1}\left(\frac{2(v-g)}{p-g+b}\right); & p-v+b\ge v-g \end{cases} \tag{A12}$$

$$E\left[\widetilde{\Pi}\left(Q^*\right)\right]=$$

$$\begin{cases} (p-g)\int_l^Q rdCr\left\{\widetilde{X}\le r\right\}-b\int_Q^m rdCr\left\{\widetilde{X}\le r\right\} & \\ \quad -b\int_n^u rdCr\left\{\widetilde{X}\le r\right\} & ;p-v+b\le v-g \\ p\int_l^m rdCr\left\{\widetilde{X}\le r\right\}-g\int_l^m rdCr\left\{\widetilde{X}\le r\right\} & \\ \quad -b\int_n^u rdCr\left\{\widetilde{X}\le r\right\} & ;p-v+b=v-g \\ rdCr\left\{\widetilde{X}\le r\right\}-b\int_Q^u rdCr\left\{\widetilde{X}\le r\right\} & \\ (p-g)\int_l^m rdCr\left\{\widetilde{X}\le r\right\} & \\ \quad +(p-g)\int_n^Q ; p-v+b\ge v-g & \end{cases} \tag{A13}$$

APPENDIX B- FUZZY SUPPLY CHAIN MODEL UNDER BUYBACK CONTRACT

Let suppose that the demand $\widetilde{X}$ is a generalized L-R type fuzzy variable with membership function $\mu_{\widetilde{X}}(x)$ as in equation (1). Here $[m, n]$ are the most likely values of fuzzy number $\widetilde{X}$; l and u are the lower and upper values respectively. Since the fuzzy demand $\widetilde{X} = (l; m; n; u)$ is a positive generalized L-R type fuzzy number, we know that the order quantity Q has three cases; that is, $Q \in [l, m]$, $Q \in [m, n]$ or $Q \in [n, u]$

By using equation (7) and equation (8); the expected value of the supplier profit function $E\left[\widetilde{\Pi}_r(Q)\right]$ of fuzzy demand $\widetilde{X}$ under buyback contract will be formulated as in (B1);

$$
\begin{aligned}
E\left[\widetilde{\Pi}_r(Q)\right] &= \int_l^Q [p * r + v_r * Q - w * Q + k * (Q - r)] dCr\left\{\widetilde{X} \le r\right\} \\
&+ \int_Q^u [p * Q - b_r * (r - Q) - v_r * Q - w * Q] dCr\left\{\widetilde{X} \le r\right\} \\
&= p \int_l^Q r dCr\left\{\widetilde{X} \le r\right\} + (v_r - w + k) Q \int_l^Q dCr\left\{\widetilde{X} \le r\right\} \\
&- k \int_l^Q r dCr\left\{\widetilde{X} \le r\right\} + (p + b_r - v_r - w) Q \int_Q^u dCr\left\{\widetilde{X} \le r\right\} \qquad \text{(B1)}
\end{aligned}
$$

The optimal order quantity Q^* will be the value that maximizes the expected value of fuzzy retailer profit function. To find the maximizing value of Q, we set;

$$\frac{dE\left[\widetilde{\Pi}_r(Q)\right]}{dQ} = 0$$

Thus, the following equations are obtained,

$$
\begin{aligned}
&\frac{dE\left[\widetilde{\Pi}_r(Q)\right]}{dQ} = p \int_l^Q r dCr\left\{\widetilde{X} \le r\right\} + (v_r - w + k) Q Cr\left\{\widetilde{X} \le Q\right\} \\
&- k \int_l^Q r dCr\left\{\widetilde{X} \le r\right\} + (p + b_r - v_r - w) Q \left(1 - Cr\left\{\widetilde{X} \le Q\right\}\right) \\
&- b_r \int_Q^u r dCr\left\{\widetilde{X} \le r\right\} \\
&0 = (k - p - b_r) * Cr\left\{\widetilde{X} \le Q\right\} + (p + b_r - v_r - w) \\
&Cr\left\{\widetilde{X} \le Q\right\} = \frac{w + v_r - p - b_r}{k - p - b_r} \qquad \text{(B2)}
\end{aligned}
$$

To find the optimum order quantity, we will determine the value of $Cr\left\{\widetilde{X} \leq Q\right\}$. The credibility of fuzzy demand with the membership function as in (1) will be as in (B3);

$$Cr\left\{\widetilde{X} \leq Q\right\} = \begin{cases} 0 & Q \leq l \\ \dfrac{L(Q)}{2} & l \leq Q \leq m \\ \dfrac{1}{2} & m \leq Q \leq n \\ 1 - \dfrac{R(Q)}{2} & n \leq Q \leq u \\ 1 & Q \geq u \end{cases} \tag{B3}$$

There are three cases to be analyzed for the value of Q in discussing the credibility value of fuzzy demand;

Case 1: $l \leq Q \leq m$

$$Cr\left\{\widetilde{X} \leq Q\right\} = \frac{w + v_r - p - b_r}{k - p - b_r} = \frac{L(Q)}{2} \tag{B4}$$

Thus, the following equations are obtained;

$$L(Q) = \frac{2(w + v_r - p - b_r)}{k - p - b_r} \text{ and } Q^* = L^{-1}\left(\frac{2(w + v_r - p - b_r)}{k - p - b_r}\right) \tag{B5}$$

Here, the value of $\frac{2(w+v_r-p-b_r)}{k-p-b_r}$ must be between $[0, 1]$, thus we can say that under the conditions of $2w + 2v_r - p \leq k + b_r$, Q^* will be in $[l, m]$. Moreover, the second derivative of $E\left[\widetilde{\Pi}(Q^*)\right]$ with respect to Q is;

$$\frac{d^2 E\left[\widetilde{\Pi}_r(Q^*)\right]}{dQ} = \frac{k - p - b_r}{2} L'(Q)$$

Since $L(Q)$is an increasing function in $[l, m]$,we find that $L'(Q) > 0$.The values of k, p and b_r are positive and $k < p + b_r$, so that we obtain $\frac{d^2 E\left[\widetilde{\Pi}_r(Q^*)\right]}{dQ} < 0$ which implies that Q^* is the optimum value which maximizes $E\left[\widetilde{\Pi}_r(Q^*)\right]$.

The expected fuzzy profit value of retailer for the optimum order quantity;

$$E\left[\widetilde{\Pi}_r(Q)\right] = (p-k)\int_l^Q rdCr\left\{\widetilde{X} \le r\right\} - b_r\left[\int_Q^m rdCr\left\{\widetilde{X} \le r\right\} + \int_n^u rdCr\left\{\widetilde{X} \le r\right\}\right] \quad \text{(B6)}$$

The expected fuzzy profit value of supplier for the optimum order quantity;

$$E\left[\widetilde{\Pi}_s(Q)\right] = k\int_l^Q rdCr\left\{\widetilde{X} \le r\right\} - b_s\left[\int_Q^m rdCr\left\{\widetilde{X} \le r\right\} + \int_n^u rdCr\left\{\widetilde{X} \le r\right\}\right] + \left[(-k-b_s)\left(\frac{w+v_r-p-b_r}{k-p-b_r}\right) + (b_s-v_s+w)\right]Q \quad \text{(B7)}$$

Case 2: $m \le Q \le n$

$$Cr\left\{\widetilde{X} \le Q\right\} = \frac{w+v_r-p-b_r}{k-p-b_r} = \frac{1}{2} \quad \text{(B8)}$$

If there exists $2w+2v_r-p=k+b_r$ case, then $E\left[\widetilde{\Pi}_r(Q)\right]$ will be maximized by any $Q \in [m,n]$. Thereby, under the condition of $2w+2v_r-p=k+b_r$, we can say that $Q^* \in [m,n]$

The expected fuzzy profit value of retailer for the optimum order quantity;

$$E\left[\widetilde{\Pi}_r(Q)\right] = (p-k)\int_l^m rdCr\left\{\widetilde{X} \le r\right\} - b_r\int_n^u rdCr\left\{\widetilde{X} \le r\right\} \quad \text{(B9)}$$

The expected fuzzy profit value of supplier for the optimum order quantity;

$$E\left[\widetilde{\Pi}_s(Q)\right] = k\int_l^m rdCr\left\{\widetilde{X} \le r\right\} - b_s\int_n^u rdCr\left\{\widetilde{X} \le r\right\} + \left[(-k-b_s)\left(\frac{w+v_r-p-b_r}{k-p-b_r}\right) + (b_s-v_s+w)\right]Q \quad \text{(B10)}$$

Case 3: $n \leq Q \leq u$

$$Cr\left\{\widetilde{X} \leq Q\right\} = \frac{w + v_r - p - b_r}{k - p - b_r} = 1 - \frac{R(Q)}{2} \tag{B11}$$

Thus, the following equations are obtained;

$$R(Q) = \frac{2(k - w - v_r)}{k - p - b_r} \text{ and } Q^* = R^{-1}\left(\frac{2(k - w - v_r)}{k - p - b_r}\right) \tag{B12}$$

Here, the value of $\frac{2(k-w-v_r)}{k-p-b_r}$ must be between $[0, 1]$, thus we can say that under the conditions of $2w + 2v_r - p \geq k + b_r$, Q^* will be in $[n, u]$. Moreover, the second derivative of $E\left[\widetilde{\Pi}_r(Q^*)\right]$ with respect to Q is;

$$\frac{d^2 E\left[\widetilde{\Pi}_r(Q^*)\right]}{dQ} = \frac{k - p - b_r}{2}\left(1 - R'(Q)\right)$$

Since $R(Q)$ is an decreasing function in $[n, u]$, we find that $\left(1 - R'(Q)\right) > 0$. The values of k, p and b_r are positive and $k < p + b_r$, so that we obtain $\frac{d^2 E\left[\widetilde{\Pi}_r(Q^*)\right]}{dQ} < 0$ which implies that Q^* is the optimum value which maximizes $E\left[\widetilde{\Pi}_r(Q^*)\right]$.

The expected fuzzy profit value of retailer for the optimum order quantity;

$$\begin{aligned} E\left[\widetilde{\Pi}_r(Q)\right] &= (p - k)\left[\int_l^m r\,dCr\left\{\widetilde{X} \leq r\right\} + \int_n^Q r\,dCr\left\{\widetilde{X} \leq r\right\}\right] \\ &\quad - b_r \int_Q^u r\,dCr\left\{\widetilde{X} \leq r\right\} \end{aligned} \tag{B13}$$

The expected fuzzy profit value of supplier for the optimum order quantity;

$$\begin{aligned} E\left[\widetilde{\Pi}_s(Q)\right] &= k\left[\int_l^m r\,dCr\left\{\widetilde{X} \leq r\right\} + \int_n^Q r\,dCr\left\{\widetilde{X} \leq r\right\}\right] - b_s \int_Q^U r\,dCr\left\{\widetilde{X} \leq r\right\} \\ &\quad + \left[(-k - b_s)\left(\frac{w + v_r - p - b_r}{k - p - b_r}\right) + (b_s - v_s + w)\right] Q \end{aligned} \tag{B14}$$

Therefore, if the demand is a generalized L-R type fuzzy number with the membership function in (1), the optimum order quantity, the optimum profits of the retailer and the supplier in fuzzy supply chain model with buyback

contract is calculated as;

$$Q^* = \begin{cases} L^{-1}\left(\dfrac{2(w+v_r-p-b_r)}{k-p-b_r}\right); & 2w+2v_r-p \le k+b_r \\ [m,n]; & 2w+2v_r-p = k+b_r \\ R^{-1}\left(\dfrac{2(k-w-v_r)}{k-p-b_r}\right); & 2w+2v_r-p \ge k+b_r \end{cases} \tag{B15}$$

$E\left[\widetilde{\Pi}_r\left(Q^*\right)\right] =$

$$\begin{cases} (p-k)\int_l^Q r dCr\left\{\widetilde{X} \le r\right\} - b_r\left[\int_Q^m r dCr\left\{\widetilde{X} \le r\right\} \right. \\ \quad \left. + \int_n^u r dCr\left\{\widetilde{X} \le r\right\}\right] & 2w+2v_r-p \le k+b_r \\ (p-k)\int_l^m r dCr\left\{\widetilde{X} \le r\right\} - b_r\int_n^u r dCr\left\{\widetilde{X} \le r\right\} & 2w+2v_r-p = k+b_r \\ (p-k)\left[\int_l^m r dCr\left\{\widetilde{X} \le r\right\} + \int_n^Q r dCr\left\{\widetilde{X} \le r\right\}\right] \\ \quad -b_r\int_Q^u r dCr\left\{\widetilde{X} \le r\right\} & 2w+2v_r-p \ge k+b_r \end{cases} \tag{B16}$$

$E\left[\widetilde{\Pi}_s\left(Q^*\right)\right] =$

$$\begin{cases} k\int_l^Q r dCr\left\{\widetilde{X} \le r\right\} - b_s\left[\int_Q^m r dCr\left\{\widetilde{X} \le r\right\} \right. \\ \quad \left. + \int_n^u r dCr\left\{\widetilde{X} \le r\right\}\right] + \left[(-k-b_s)\left(\frac{w+v_r-p-b_r}{k-p-b_r}\right)\right. & 2w+2v_r-p \le k+b_r \\ \quad \left. +(b_s-v_s+w)\right]Q; \\ k\int_l^m r dCr\left\{\widetilde{X} \le r\right\} - b_s\int_n^u r dCr\left\{\widetilde{X} \le r\right\} & 2w+2v_r-p = k+b_r \\ \quad +\left[(-k-b_s)\left(\frac{w+v_r-p-b_r}{k-p-b_r}\right)+(b_s-v_s+w)\right]Q; \\ k\left[\int_l^m r dCr\left\{\widetilde{X} \le r\right\} + \int_n^Q r dCr\left\{\widetilde{X} \le r\right\}\right] \\ \quad -b_s\int_Q^U r dCr\left\{\widetilde{X} \le r\right\} + \left[(-k-b_s)\left(\frac{w+v_r-p-b_r}{k-p-b_r}\right)\right. & 2w+2v_r-p \ge k+b_r \\ \quad \left. +(b_s-v_s+w)\right]Q; \end{cases} \tag{B17}$$

APPENDIX C- FUZZY SUPPLY CHAIN MODEL UNDER REVENUE SHARING CONTRACT

Let suppose that the demand $\widetilde{X}$ is a generalized L-R type fuzzy variable with membership function $\mu_{\widetilde{X}}(x)$ as in equation (1). Here $[m, n]$ are the most likely values of fuzzy number $\widetilde{X}$; l and u are the lower and upper values respectively. Since the fuzzy demand $\widetilde{X} = (l; m; n; u)$ is a positive generalized L-R type fuzzy number, we know that the order quantity Q has three cases; that is, $Q \in [l, m]$, $Q \in [m, n]$ or $Q \in [n, u]$

By using equation (7) and equation (8); the expected value of the total profit function $E\left[\widetilde{\Pi}_r(Q)\right]$ of fuzzy demand $\widetilde{X}$ under revenue sharing contract will be formulated as in (C1);

$$E\left[\widetilde{\Pi}_r(Q)\right] = \int_l^Q [p * \emptyset * r + g * \emptyset * (Q - r) - v_r * Q - w * Q]$$

$$dCr\left\{\widetilde{X} \le r\right\} + \int_Q^u [p * \emptyset * Q - b_r * (r - Q) - v_r * Q - w * Q] dCr\left\{\widetilde{X} \le r\right\}$$

$$(p - g) * \emptyset * \int_l^Q r dCr\left\{\widetilde{X} \le r\right\} + (g * \emptyset - v_r - w) Q \int_l^Q dCr\left\{\widetilde{X} \le r\right\}$$

$$-b_r \int_Q^u r dCr\left\{\widetilde{X} \le r\right\} + (p * \emptyset + b_r - v_r - w) Q \int_Q^u dCr\left\{\widetilde{X} \le r\right\} \qquad \text{(C1)}$$

The optimal order quantity Q^* will be the value that maximizes the expected value of fuzzy retailer profit function. To find the maximizing value of Q, we set;

$$\frac{dE\left[\widetilde{\Pi}_r(Q)\right]}{dQ} = 0$$

Thus, the following equations are obtained,

$$\frac{dE\left[\widetilde{\Pi}_r(Q)\right]}{dQ} = (p - g) * \emptyset * \int_l^Q r dCr\left\{\widetilde{X} \le r\right\}$$

$$+ (g * \emptyset - v_r - w) Q Cr\left\{\widetilde{X} \le Q\right\} - b_r \int_Q^u r dCr\left\{\widetilde{X} \le r\right\}$$

$$+ (p * \emptyset + b_r - v_r - w) Q\left(1 - Cr\left\{\widetilde{X} \le Q\right\}\right)$$

$$0 = (g * \emptyset - p * \emptyset - b_r) * Cr\{X \le Q\} + (p * \emptyset + b_r - v_r - w)$$

$$Cr\left\{\widetilde{X} \le Q\right\} = \frac{w + v_r - p\emptyset - b_r}{g * \emptyset - p * \emptyset - b_r} \qquad \text{(C2)}$$

To find the optimum order quantity, we will determine the value of $Cr\left\{\widetilde{X} \leq Q\right\}$. The credibility of fuzzy demand with the membership function as in (1) will be as in (C3);

$$Cr\left\{\widetilde{X} \leq Q\right\} = \begin{cases} 0 & Q \leq l \\ \frac{L(Q)}{2} & l \leq Q \leq m \\ \frac{1}{2} & m \leq Q \leq n \\ 1 - \frac{R(Q)}{2} & n \leq Q \leq u \\ 1 & Q \geq u \end{cases} \tag{C3}$$

There are three cases to be analyzed for the value of Q in discussing the credibility value of fuzzy demand;

Case 1: $l \leq Q \leq m$

$$Cr\left\{\widetilde{X} \leq Q\right\} = \frac{w + v_r - p\emptyset - b_r}{g * \emptyset - p * \emptyset - b_r} = \frac{L(Q)}{2} \tag{C4}$$

Thus, the following equations are obtained;

$$L(Q) = \frac{2 * (w + v_r - p\emptyset - b_r)}{g * \emptyset - p * \emptyset - b_r} \text{ and } Q^* = L^{-1}\left(\frac{2 * (w + v_r - p\emptyset - b_r)}{g * \emptyset - p * \emptyset - b_r}\right) \tag{C5}$$

Here, the value of $\frac{2*(w+v_r-p\emptyset-b_r)}{g*\emptyset-p*\emptyset-b_r}$ must be between $[0, 1]$, thus we can say that under the conditions of $2w + 2v_r - p\emptyset \leq g\emptyset + b_r$, Q^* will be in $[l, m]$. Moreover, the second derivative of $E\left[\widetilde{\Pi}_r(Q^*)\right]$ with respect to Q is;

$$\frac{d^2 E\left[\widetilde{\Pi}_r(Q^*)\right]}{dQ} = \frac{(g * \emptyset - p * \emptyset - b_r)}{2} L'(Q)$$

Since $L(Q)$ is an increasing function in $[l, m]$, we find that $L'(Q) > 0$. The values of g, p, $\emptyset$ and b_r are positive and $g * \emptyset < p * \emptyset + b_r$, so that we obtain $\frac{d^2 E\left[\widetilde{\Pi}_r(Q^*)\right]}{dQ} < 0$ which implies that Q^* is the optimum value which maximizes $E\left[\widetilde{\Pi}_r(Q^*)\right]$.

The expected fuzzy profit value of retailer for the optimum order quantity;

$$\begin{aligned} E\left[\widetilde{\Pi}_r(Q)\right] \;=\; & (p - g)\emptyset \int_l^Q r\, dCr\left\{\widetilde{X} \leq r\right\} \\ & -b_r\left[\int_Q^m r\, dCr\left\{\widetilde{X} \leq r\right\} + \int_n^u r\, dCr\left\{\widetilde{X} \leq r\right\}\right] \end{aligned} \tag{C6}$$

The expected fuzzy profit value of supplier for the optimum order quantity;

$$E\left[\widetilde{\Pi}_s(Q)\right] = (p-g)(1-\emptyset)\int_l^Q r\,dCr\left\{\widetilde{X}\le r\right\} - b_s\left[\int_Q^m r\,dCr\left\{\widetilde{X}\le r\right\} + \int_n^u r\,dCr\left\{\widetilde{X}\le r\right\}\right] + \left[(-b_s-(p-g)(1-\emptyset))\left(\tfrac{w+v_r-p-b_r}{k-p-b_r}\right) + (b_s-v_s+w+(1-\emptyset)\,p)\right]Q \quad \text{(C7)}$$

Case 2: $m \le Q \le n$

$$Cr\left\{\widetilde{X}\le Q\right\} = \frac{w+v_r-p\emptyset-b_r}{g*\emptyset-p*\emptyset-b_r} = \frac{1}{2} \quad \text{(C8)}$$

If there exists $2w+2v_r-p\emptyset = g\emptyset+b_r$ case, then $E\left[\widetilde{\Pi}_r(Q)\right]$ will be maximized by any $Q\in[m,n]$. Thereby, under the condition of $2w+2v_r-p\emptyset = g\emptyset+b_r$, we can say that $Q^*\in[m,n]$

The expected fuzzy profit value of retailer for the optimum order quantity;

$$E\left[\widetilde{\Pi}_r(Q)\right] = (p-g)\emptyset\int_l^m r\,dCr\left\{\widetilde{X}\le r\right\} - b_r\int_n^u r\,dCr\left\{\widetilde{X}\le r\right\} \quad \text{(C9)}$$

The expected fuzzy profit value of supplier for the optimum order quantity;

$$E\left[\widetilde{\Pi}_s(Q)\right] = (p-g)(1-\emptyset)\int_l^m r\,dCr\left\{\widetilde{X}\le r\right\} - b_s\int_n^u r\,dCr\left\{\widetilde{X}\le r\right\} + \left[(-b_s-(p-g)(1-\emptyset))\left(\tfrac{w+v_r-p-b_r}{k-p-b_r}\right) + (b_s-v_s+w+(1-\emptyset)\,p)\right]Q \quad \text{(C10)}$$

Case 3: $n \le Q \le u$

$$Cr\left\{\widetilde{X}\le Q\right\} = \frac{w+v_r-p\emptyset-b_r}{g*\emptyset-p*\emptyset-b_r} = 1-\frac{R(Q)}{2} \quad \text{(C11)}$$

Thus, the following equations are obtained;

$$R(Q) = \frac{2(g\emptyset-w-v_r)}{g*\emptyset-p*\emptyset-b_r} \text{ and } Q^* = R^{-1}\left(\frac{2(g\emptyset-w-v_r)}{g*\emptyset-p*\emptyset-b_r}\right) \quad \text{(C12)}$$

Here, the value of $\frac{2(g\emptyset-w-v_r)}{g*\emptyset-p*\emptyset-b_r}$ must be between $[0,1]$, thus we can say that under the conditions of $2w+2v_r-p\emptyset \ge g\emptyset+b_r$, Q^* will be in $[n,u]$.

Moreover, the second derivative of $E\left[\widetilde{\Pi}_r(Q^*)\right]$ with respect to Q is;

$$\frac{d^2E\left[\Pi_r(Q^*)\right]}{dQ} = \frac{(g*\emptyset - p*\emptyset - b_r)}{2}\left(1 - R^{'}(Q)\right)$$

Since $R(Q)$ is an decreasing function in $[n,u]$, we find that $\left(1-R^{'}(Q)\right) > 0$. The values of g, $p, \emptyset$ and b_r are positive and $g*\emptyset < p*\emptyset + b_r$, so that we obtain $\frac{d^2E\left[\widetilde{\Pi}_r(Q^*)\right]}{dQ} < 0$ which implies that Q^* is the optimum value which maximizes $E\left[\widetilde{\Pi}_r(Q^*)\right]$.

The expected fuzzy profit value of retailer for the optimum order quantity;

$$\begin{aligned} E\left[\widetilde{\Pi}_r(Q)\right] &= (p-g)\emptyset\left[\int_l^m r\,dCr\left\{\widetilde{X}\le r\right\} + \int_n^Q r\,dCr\left\{\widetilde{X}\le r\right\}\right] \\ &\quad -b_r\int_Q^u r\,dCr\left\{\widetilde{X}\le r\right\} \end{aligned} \tag{C13}$$

The expected fuzzy profit value of supplier for the optimum order quantity;

$$\begin{aligned} &E\left[\widetilde{\Pi}_s(Q)\right] = \\ &(p-g)(1-\emptyset)\left[\int_l^m r\,dCr\left\{\widetilde{X}\le r\right\} + \int_n^Q r\,dCr\left\{\widetilde{X}\le r\right\}\right] \\ &-b_s\int_Q^U r\,dCr\left\{\widetilde{X}\le r\right\} + \left[(-b_s-(p-g)(1-\emptyset))\left(\tfrac{w+v_r-p-b_r}{k-p-b_r}\right)\right. \\ &\left.+(b_s-v_s+w+(1-\emptyset)p)\right]Q \end{aligned} \tag{C14}$$

Therefore, if the demand is a generalized L-R type fuzzy number with the membership function in (1),the optimum order quantity, the optimum profits of the retailer and the supplier in fuzzy supply chain model with revenue sharing contract is calculated as;

$$Q^* = \begin{cases} L^{-1}\left(\dfrac{2*(w+v_r-p\emptyset-b_r)}{g*\emptyset-p*\emptyset-b_r}\right); & 2w+2v_r-p\emptyset \ge g\emptyset+b_r \\ [m,n]; & 2w+2v_r-p\emptyset \ge g\emptyset+b_r \\ R^{-1}\left(\dfrac{2(g\emptyset-w-v_r)}{g*\emptyset-p*\emptyset-b_r}\right); & 2w+2v_r-p\emptyset \ge g\emptyset+b_r \end{cases} \tag{C15}$$

$$E\left[\widetilde{\Pi}_r\left(Q^*\right)\right] =$$

$$\begin{cases} (p-g)\emptyset \int_l^Q r dCr\left\{\widetilde{X} \le r\right\} - b_r\left[\int_Q^m r dCr\left\{\widetilde{X} \le r\right\}\right. & \\ \quad \left. + \int_n^u r dCr\left\{\widetilde{X} \le r\right\}\right] & 2w + 2v_r - p\emptyset \ge g\emptyset + b_r \\ (p-g)\emptyset \int_l^m r dCr\left\{\widetilde{X} \le r\right\} - b_r \int_n^u r dCr\left\{\widetilde{X} \le r\right\} & 2w + 2v_r - p\emptyset \ge g\emptyset + b_r \\ (p-g)\emptyset\left[\int_l^m r dCr\left\{\widetilde{X} \le r\right\} + \int_n^Q r dCr\left\{\widetilde{X} \le r\right\}\right] & \\ \quad -b_r \int_Q^u r dCr\left\{\widetilde{X} \le r\right\} & 2w + 2v_r - p\emptyset \ge g\emptyset + b_r \end{cases} \quad \text{(C16)}$$

$$E\left[\widetilde{\Pi}_s\left(Q^*\right)\right] =$$

$$\begin{cases} (p-g)(1-\emptyset) \int_l^Q r dCr\left\{\widetilde{X} \le r\right\} & \\ \quad -b_s\left[\int_Q^m r dCr\left\{\widetilde{X} \le r\right\} + \int_n^u r dCr\left\{\widetilde{X} \le r\right\}\right] & 2w + 2v_r - p \le k + b_r \\ \quad + \left[(-b_s - (p-g)(1-\emptyset))\left(\frac{(w+v_r-p\emptyset-b_r)}{g*\emptyset-p*\emptyset-b_r}\right)\right. & \\ \quad \left. + (b_s - v_s + w + (1-\emptyset)\, p)\right] Q; & \\ (p-g)(1-\emptyset) \int_l^m r dCr\left\{\widetilde{X} \le r\right\} - b_s \int_n^u r dCr\left\{\widetilde{X} \le r\right\} & 2w + 2v_r - p = k + b_r \\ + \left[(-b_s - (p-g)(1-\emptyset))\left(\frac{(w+v_r-p\emptyset-b_r)}{g*\emptyset-p*\emptyset-b_r}\right)\right. & \\ \quad \left. + (b_s - v_s + w + (1-\emptyset)\, p)\right] Q; & \\ (p-g)(1-\emptyset)\left[\int_l^m r dCr\left\{\widetilde{X} \le r\right\} + \int_n^Q r dCr\left\{\widetilde{X} \le r\right\}\right] & 2w + 2v_r - p \ge k + b_r \\ -b_s \int_Q^U r dCr\left\{\widetilde{X} \le r\right\} + \left[(-b_s - (p-g)(1-\emptyset))\left(\frac{(w+v_r-p\emptyset-b_r)}{g*\emptyset-p*\emptyset-b_r}\right)\right. & \\ \left. + (b_s - v_s + w + (1-\emptyset)\, p)\right] Q; & \end{cases}$$

(C17)