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Combined outputs variance constrained and input variance constrained design for flight control

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Abstract

In this article, an algorithm combining two different variance constrained control strategies (i.e. *o*utput *v*ariance *c*onstrained: OVC controller and *i*nput *v*ariance *c*onstrained: IVC controller) is developed. Performance of the resulting novel control technique called as combined output and input variance constrained controller is evaluated on helicopters. Their linearized state-space models are used to see feasibility and efficiency of this new strategy (i.e., output and input variance constrained). Closed loop responses of some outputs and inputs of interest and also some outputs out of interest are investigated while using output and input variance constrained for air vehicle *f*light *c*ontrol system. Then, closed loop performance of output and input variance constrained is compared with respect to the two other classical control techniques (i.e. outputs variance constrained and *i*nput *v*ariance *c*onstrained). Finally, robustness of output and input variance constrained with respect to the modeling uncertainities is examined.

Keywords

Constrained control, variance constrained control, OVC and IVC, flight control, helicopters

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Introduction

Throughout the years, many control strategies for air vehicle *f*light *c*ontrol *s*ystem (FCS) design have been extensively researched. In historical sequence, some of these methods are classical pole placement and simple feedback methods,[1–3] modified linear quadratic regulator and *l*inear *q*uadratic *G*aussian (LQG) controllers,[4–6] modified H_∞ control synthesis,[7–9] constrained *m*odel *p*redictive *c*ontrol,[10–12] and variance-constrained controllers.[13–21] Variance-constrained controllers are one of these control techniques. They have many advantages *w*ith *r*espect *t*o (w.r.t.) the other control strategies existing in the literature. First of all, these controllers are modified LQG controllers and they use Kalman filters as state estimators. Secondly, these controllers employ second-order information (i.e., state covariance matrix, see Skelton[22] and Skelton et al.[23] for more information) and this type of information is very useful during multivariable control system design since all stabilizing controllers are parameterized in relation to the physically meaningful state covariance matrix. Finally, for large and strongly coupled *m*ulti-*i*nput, *m*ulti-*o*utput systems similar with the ones met in air vehicles control, variance-constrained control techniques give guarantees on the transient behavior of independent variables via enforcing upper limits on the variance of these variables.

Variance-constrained controllers have been used for many aerospace vehicles (e.g., helicopters;[13–21] tiltrotor aircraft;[22] Hubble space telescope;[24] and tensegrity structures[25,26]) before. For example, in Oktay and Sultan[14] variance-constrained controllers were used for helicopter FCS during maneuvers (i.e., level banked turn and helical turn). In this study, their performance was also evaluated when some of the helicopter sensors failed. Satisfactory results (meaning that variance constraints on outputs–inputs were satisfied and also closed loop systems were exponentially stabilized) were obtained for helicopter FCS. Robustness of the closed loop systems (obtained by integration of helicopter linearized state-space model and specific variance constrained controller) w.r.t. the modeling uncertainties (i.e., flight condition variation and helicopter inertial parameters variation) was also researched and it was seen that these controllers had

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stability robustness. Moreover, in Oktay and Sultan,[15] they were used for passively morphing helicopters. In their study, the passively morphing parameters were blade chord length, blade flapping spring stiffness, blade linear mass density, blade length, blade twist, and main rotor angular speed. In that study, a stochastic optimization method (i.e., *s*imultaneous *p*erturbation *s*tochastic *a*pproximation, SPSA) was used in order to simultaneously design helicopter and FCS (i.e., outputs variance constrained [OVC]). For the 40 kt straight-level flight condition with only 5% changes on passive design parameters, around 30% of the FCS energy was saved. It is important to note that for this study, the design parameters of redesigned helicopter obtained simultaneous design idea, which do not change during flight (i.e., straight-level flight or maneuvering flight). In another study of Oktay and Sultan,[17] variance-constrained controllers (i.e. OVCs) were used for actively morphing helicopters. In this study, the morphing parameters were blade chord length, blade length, blade twist, and main rotor angular speed. And also in this study using SPSA method, helicopter design parameters and FCS were simultaneously redesigned. For the 40 kt straight-level flight with only 5% changes on active design parameters, around 85% of the FCS energy was saved. The main difference between this and previous studies is that in the active case, the helicopter design parameters except helicopter FCS parameters change during flight, but in prescribed interval ($\pm 5\%$). On the other hand, in the passive case, the helicopter design parameters and helicopter FCS parameters do not change during flight (i.e., straight-level flight or maneuvering flight).

Variance-constrained controllers specifically OVC and *i*nput *v*ariance *c*onstrained (IVC) strategies have been applied on aerospace vehicles (e.g., helicopters, tiltrotor aircraft, Hubble space telescope, and tensegrity structures) before. However, they have never been used in such a way that output and variance constraints are considered together. However, it is also crucial to build a control strategy dealing with variance constraints on outputs and inputs simultaneously since many engineering systems are face to face with constraints on both outputs and inputs. In this article, this idea is *first time* considered and a successful algorithm for this consideration is developed. Since output and input variance constraints (O&I-VC) are strongly related to each other (it is significant to note that this relation is not linear and it is highly nonlinear), it is sometimes impossible to develop a controller satisfying strict variance constraints on outputs and inputs simultaneously. This problem creates a new idea called as possible achievable output and input variance intervals. In this article, this feasibility study is also *first time* applied.

This article first presents definition of classical variance-constrained control strategies (i.e., OVC and IVC). And the development of new control strategy (i.e., combined OVC and IVC, abbreviated with O&I-VC) is explained in detail. Then feasibility of O&I-VC is discussed. After discussing FCS design, mathematical models of air vehicle (i.e., helicopters) used for the performance evaluation of O&I-VC are introduced. Finally, using linearized state-space helicopter models, closed loop responses of some outputs and inputs of interest for FCS design, and also some outputs out of interest for FCS design are deeply examined and discussed. Then, robustness of O&I-VC w.r.t. the modeling uncertainities is examined.

Definition of classical variance constrained control strategies

Many engineering systems and aerospace vehicles are always exposed to output and input limitations. Moreover, minimization of control energy is generally a key requirement. For example, large Euler angle deviations of civil helicopters from the trim position are not desirable for the safety and comfort reasons. In addition to this, control energy is produced via fuel consumption, and energy minimization is equivalent with fuel consumption minimization. In both situations, minimization of control energy helps to weight reduction of any aerospace vehicle system. Therefore, variance-constrained controllers have been developed to fulfill these requirements for aerospace vehicles. Specifically OVC[27,28] and input variance-constrained control (IVC)[28] have been used for this purpose in recent years. These controllers guarantee satisfaction of output (i.e., OVC) or input (i.e., IVC) variance constraints. Their brief definitions are given subsequently.

Output variance-constrained control (OVC)

For a given continuous *l*inear *t*ime *i*nvariant (LTI), stabilizable and detectable system

$$\dot{x}_p = A_p x_p + B_p u_p + w_p, \quad y = C_p x_p, \quad z = M_p x_p + v \tag{1}$$

and a positive definite input weighting matrix, $R > 0$, find a full-order dynamic controller

$$\dot{x}_c = A_c x_c + Fz, \quad u_p = G x_c \tag{2}$$

to solve the problem

$$\min_{A_c, F, G} J = E_\infty u_p^T R u_p = tr(RGX_{c_j}G^T) \tag{3}$$

subject to variance constraints on the output/outputs

$$E_\infty y_i^2 \leqslant \sigma_i^2, \; i = 1, \ldots, n_y \tag{4}$$

Above y and z represents outputs of interest and sensor measurements, respectively, w_p and $\boldsymbol{v}$ are zero-mean uncorrelated Gaussian white noises with intensities of $\boldsymbol{W}$ and $\boldsymbol{V}$, respectively, F and G are state estimator and controller gain matrices, respectively, x_c is the controller state vector, σ_i^2 is the upper limit imposed on the ith output variance, n_y is the number of outputs, $E_\infty \triangleq lim_{t\to\infty}E$, and E is the expectation operator. Furthermore, tr and T denote matrix trace and matrix transpose operators, respectively. The quantity of J generally called as FCS energy or FCS cost and it is computed using also the state covariance matrix, X_{c_j}.

In reality, OVCs are enhanced LQGs since they guarantee satisfaction of variance constraints on output/outputs of interest. OVC solution reduces to an LQG problem solution via output penalty matrix $\boldsymbol{Q}>0$ depending on the inequality constraints. An algorithm for the selection of $\boldsymbol{Q}$ is presented in Hsieh et al.[27] and Zhu and Skelton.[28] After the algorithm converges and the output penalty matrix $\boldsymbol{Q}$ is obtained, OVC parameters are

$$A_c = A_p + B_pG - FM_p, \quad F = XM_p^TV^{-1}, \quad G = -R^{-1}B_p^TK \tag{5}$$

Here, $\boldsymbol{X}$ and $\boldsymbol{K}$ are solutions of solutions of two algebraic Riccati equations

$$0 = XA_p^T + A_pX - XM_p^TV^{-1}M_pX + W \tag{6a}$$

$$0 = KA_p + A_p^TK - KB_pR^{-1}B_p^TK + C_p^TQC_p \tag{6b}$$

Input variance-constrained control (IVC)

IVC problem is fundamentally the dual of OVC: for a given continuous LTI, stabilizable and detectable system (see equation (1)) and a given positive definite output penalty matrix, $Q>0$, a full-order dynamic controller (see equation (2)) must be obtained to solve

$$\min_{A_c,F,G} E_\infty y^TQy \tag{7}$$

subject to

$$E_\infty u_{pi}^2 \leq \mu_i^2, \quad i = 1,\ldots,n_{u_p} \tag{8}$$

where μ_i^2 is the upper variance limit on the ith input and n_{u_p} is the number of inputs. IVC solution reduces to an LQG problem solution via choosing $R>0$. An algorithm for the selection of R is presented in Zhu and Skelton.[28] It is important to note that OVC explicitly minimizes control energy. However, IVC reduces this energy via the inequality variance constraints on the inputs of interest. Compared to LQG where weighting matrices (i.e. $\boldsymbol{Q}$ and $\boldsymbol{R}$), are selected in ad hoc manner, IVC supply an intelligent way of choosing these penalty matrices, which guarantees constraint satisfaction.

Combined OVC and IVC (i.e., O&I-VC)

OVC and IVC techniques have been never used such a method that output and variance constraints are considered simultaneously and both of them are applied. However, it is significant to design a control technique considering variance constraints on outputs and inputs simultaneously, because most the engineering systems have constraints on both outputs and inputs. Therefore, a method dealing both outputs and inputs is vital. The O&I-VC technique is developed in order to fulfill this gap. It is applied using MATLAB software. The required input data to apply O&I-VC technique are $\{A_p, B_p, C_p, D_p, M_p\}$, $\{\boldsymbol{W}, \boldsymbol{V}\}$, $\{Q_0, R_0\}$, $\{\sigma^2, \mu^2\}$, and $\{\xi_{\text{OVC}}, \xi_{\text{IVC}}, \xi_{\text{O\&I-VC}}\}$. Here matrices of $\{A_p, B_p, C_p, D_p, M_p\}$ are linearized state-space model parameters, $\{\boldsymbol{W}, \boldsymbol{V}\}$ are process and measurement noise parameters, $\{Q_0, R_0\}$ are initial values of weighting matrices, $\{\sigma^2, \mu^2\}$ are variance constraints for OVC and IVC, and $\{\xi_{\text{OVC}}, \xi_{\text{IVC}}, \xi_{\text{O\&I-VC}}\}$ are stopping criteria for OVC, IVC, and O&I-VC algorithms. The schematic of O&I-VC technique is given in Figure 1 (see Hsieh et al.[27] and Zhu and Skelton[28] for OVC algorithm and see Zhu and Skelton[28] for IVC algorithm, see also Appendix for brief algorithms of these techniques).

For the O&I-VC algorithm, first OVC is applied using linearized state-space model matrices, noise parameters, initial values of weighting matrices, its constraint parameters, and its stopping criterion. After running OVC algorithm, the resulting weighting matrices (i.e., Q_i and R_{i-1}) are found where i is the ith iteration. Then using resulting weighting matrices,

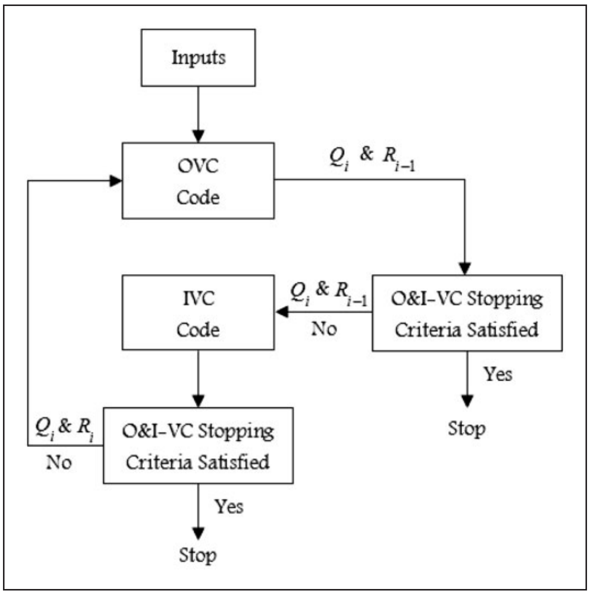

Figure 1. Schematic of O&I-VC algorithm. *Note*: O&I-VC, output and input variance constraints; OVC, outputs variance constrained; IVC, input variance constrained.

expected values of inputs of interest (i.e., $E_\infty u_{pi}^2, i = 1, \ldots, ., n_{u_p}$) are evaluated. After that stopping box of O&I-VC algorithm is visited using expected values of outputs and inputs of interest. If it is satisfied, the O&I-VC algorithm is stopped. Otherwise, using same linearized state-space model matrices and noise parameters, its constraints, its own stopping criterion, and obtained weighting matrices (it is crucial to note that only $\boldsymbol{Q}$ matrix changes during running OVC algorithm), IVC is applied. After running IVC code, the resulting weighting parameters (i.e., Q_i and R_i) are found. Then using resulting weighting matrices, expected values of outputs of interest ($E_\infty y_i^2, i = 1, \ldots, n_y$) are evaluated. After that stopping box of O&I-VC is again visited using expected values of outputs and inputs of interest. If it is satisfied, the O&I-VC algorithm is stopped. Otherwise, OVC is again applied. This loop is repeated until both O&I-VCs are satisfied.

The stopping box of O&I-VC algorithm checks that $E_\infty y_i^2 - \xi_{\mathrm{O\&I-VC}} \leqslant \sigma_i^2, i = 1, \ldots, n_y$ after applying OVC and $E_\infty u_{pi}^2 - \xi_{\mathrm{O\&I-VC}} \leqslant \mu_i^2, i = 1, \ldots, ., n_{u_p}$ after applying IVC. The stopping criterion $\xi_{\mathrm{O\&I-VC}}$ is

$$\xi_{\mathrm{O\&I-VC}} = \begin{cases} 2\xi_{\mathrm{IVC}} \text{ after applying OVC} \\ 2\xi_{\mathrm{OVC}} \text{ after applying IVC} \end{cases} \tag{9}$$

It is important to emphasize that these constraints are only satisfied in a feasible range. Therefore, a feasibility study of O&I-VC is done next.

Feasibility of O&I-VC

For any physical or engineering system, behaviors of outputs and inputs are always related. This relation may be linear, nonlinear, exponential, or in a different way. Due to this relation, it is not always guaranteed to satisfy output and input constraints simultaneously. Therefore, it is also not guaranteed to satisfy desired output and input variance limitations at the same time. In this section, for this reason, feasibility of O&I-VC is discussed.

For the Puma SA 330 helicopter, linearized around 40 kt straight-level flight condition, O&I-VCs is designed with parametric output and input constraints of $\sigma^2_{\phi_A,\theta_A} = a * 10^{-4}\begin{bmatrix}1 & 1\end{bmatrix}$ and $\mu^2_{\theta_o,\theta_c,\theta_s,\theta_T} = b * 10^{-4}\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}$. The nonfeasible region is shown in Figure 2. For the nonfeasible region, it is impossible the satisfy bounds of demanded O&I-VC. For other region in this figure, O&I-VC can be easily designed. From this figure, it can be easily concluded that O&I-VC can be designed only for reasonable constraints.

Air vehicle models

In this article, the helicopter models that are generated and presented in Oktay[13] and Oktay and Sultan[29] in detail and briefly summarized here are employed. In summary, physics principles are applied and this leads to dynamic models comprised of finite set of ordinary differential equations (i.e., ODEs). These models are very advantageous for control system design since they simplify the direct use of modern control theory. Modern control theory depends on state space demonstration of any system's dynamics and it is easily found from ODEs. The main modeling assumptions are given next. First of all, multibody system approach is used to model helicopter and all typical helicopter subsystem (i.e., fuselage, empennage: tail rotor hub and shaft, and horizontal tailplane). Second, Pitt–Peter's static inflow model is used for the main rotor downwash. However, for the modeling of fuselage, an analytical aerodynamics formulation is applied. The modeling methodology explained here was implemented using Maple software and fairly complex nonlinear helicopter model with 28 equations (i.e., 9 fuselage equations, 16 blade flapping and lead-lagging equations, and 3 static main rotor downwash equations). After simplifying (it is important to note that a systematic model simplification approach is used to decrease number of terms in these nonlinear equations, see Oktay[13] and Oktay and

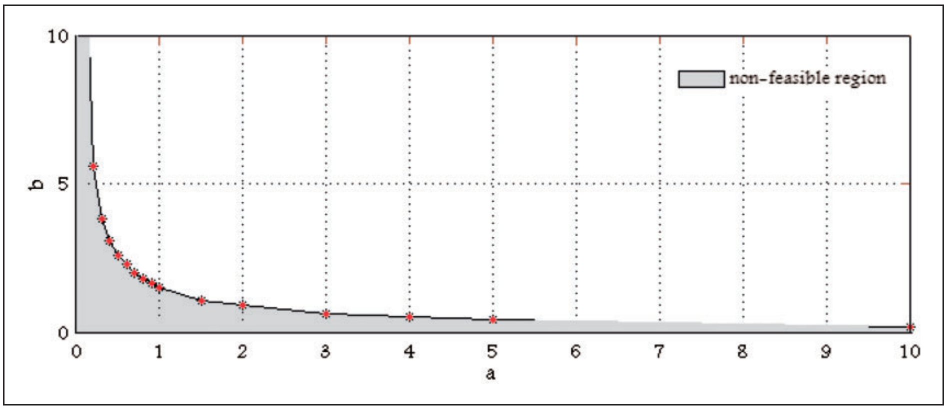

Figure 2. Feasibility results for O&I-VC on Puma SA 330 helicopter.

Sultan[29] for more details) and trimming the nonlinear model, continuous LTI systems are obtained.

$$\dot{x}_p = A_p x_p + B_p u_p \tag{10}$$

Here x_p and u_p are perturbed state and control vectors and matrices A_p and B_p are of size 25×25 and 25×4. There are 25 perturbed states. These are nine fuselage states (i.e., three linear velocity, three angular velocity states, and three Euler angle states), eight blade flapping states and eight blade lead-lagging states, and four controls (i.e., three main rotor controls: collective, θ_0, lateral cyclic, θ_c, and longitudinal cyclic θ_s, and one tail rotor control: collective, θ_T). It is important to note that Puma SA 330 helicopter was used to validate helicopter models employed in this article (see Oktay[13] and Padfield[30] for technical details of this helicopter).

Results

Application of O&I-VC on helicopters

It is first required to note that for all of the numerical results reported in this article, the sensor measurements (z in equation (1)) were helicopter linear velocities, angular velocities, and Euler angles and the outputs of interest (y in equation (1)) were helicopter roll and pitch angles. The inputs of interest were main rotor blade pitch angles and collective tail rotor angle. The O&I-VCs were $\sigma^2_{\phi_A,\theta_A} = 0.1 * 10^{-4}\begin{bmatrix} 1 & 1 \end{bmatrix}$ and $\mu^2_{\theta_o,\theta_c,\theta_s,\theta_T} = 1.2 * 10^{-4}\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$. The stopping criteria used for O&I-VC algorithm was $\xi_{\mathrm{OVC}} = 10^{-6}\begin{bmatrix} 1 & 1 \end{bmatrix}$ and $\xi_{\mathrm{IVC}} = 1.2 * 10^{-5}\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$. The $\xi_{\mathrm{O\&I-VC}}$ is calculated using equation (9). The *nondimensionalized* noise intensities were considered $W = 10^{-7} \times I_{25}$ and $V = 10^{-7} \times I_9$ where I_n is the identity matrix.

Closed loop simulations

In order to better examine the benefits of O&I-VC w.r.t. the other two existing classical variance-constrained control techniques (i.e., OVC and IVC), closed loop performance of helicopter is investigated using OVC, IVC, and O&I-VC, individually. For this purpose, a helicopter model linearized around 40 kt straight-level flight condition is used. For the discussions given subsequently, closed loop system, which is obtained by integration of helicopter and O&I-VC designed for it, is referred as the first closed loop system. Similarly, closed loop system, which is obtained by integration of helicopter and OVC designed for it, is referred as the second closed loop system. Moreover, closed loop system, which is obtained by integration of helicopter and IVC designed for it, is referred as the third closed loop system. One more closed loop is created in order to check stability robustness of O&I-VC w.r.t. the modeling uncertainities (i.e., variation of flight condition and all helicopter inertial parameters). For this purpose, O&I-VC designed for 40 kt straight-level flight condition is evaluated on the helicopter model linearized around 50 kt straight-level flight condition and experiencing 10% reduction in all helicopter inertial parameters. This closed loop system is referred as fourth closed loop system. Since the controller has no information for this uncertainty, it is called as unaware one.

In next set of figures, degrees are used to better demonstrate the behaviors of specific variables. The labels of O&I-VC, OVC, and IVC are referring to the first, second, and third closed loop systems, respectively. It is also important to note that since linearized models are used, variables correspond to *perturbations* from their trim values in all next figures. Moreover, all of these three controllers exponentially stabilize the nominal closed loop systems.

In Figure 3, closed loop responses of outputs of interest (i.e., helicopter roll and pitch angles) and some of inputs of interest (i.e., main rotor longitudinal cyclic blade pitch angle and collective tail rotor angle) are given when first closed loop system (solid black line) and second closed loop system (dotted black line) are all excited by white noise perturbations. From Figure 3, it can be concluded that peak values of most of helicopter control inputs (e.g., θ_s and θ_T) considerably reduce when O&I-VC is used rather than OVC. Moreover, peak values of outputs of interest (i.e. ϕ_A and θ_A) are almost the same using any of the O&I-VC and OVC methods. These simulation results indicate that O&I-VC is more advantageous than OVC since it reduces peak values of control inputs while fixing peak values of outputs of interest. This means that same output performance can be satisfied with less control energy.

In Figure 4, closed loop responses of some of the inputs of interest (i.e., longitudinal and lateral main rotor blade pitch angles) and one of the outputs of interest (i.e., helicopter pitch angle) are given when first closed loop system (solid black line) and third closed loop system (dotted black line) are both excited by white noise perturbations. From Figure 4, it can be easily seen that peak values of outputs of interest (e.g., θ_A) considerably reduce when O&I-VC is used rather than IVC. Peak values of inputs of interest (e.g., θ_c and θ_s) are almost the same using any of the O&I-VC and IVC methods. These simulation results indicate that O&I-VC is more advantageous than IVC since it reduces peak values of outputs of interest while fixing peak values of helicopter control inputs. This means that same input performance can be satisfied with less peak values of outputs of interest.

In Figure 5, closed loop responses of some of helicopter fuselage states and blade states (β_0, ζ_0: collective blade flapping and lead-lagging angles; u, w: longitudinal and vertical linear velocities; q, r: lateral and vertical angular velocities) are given for the first

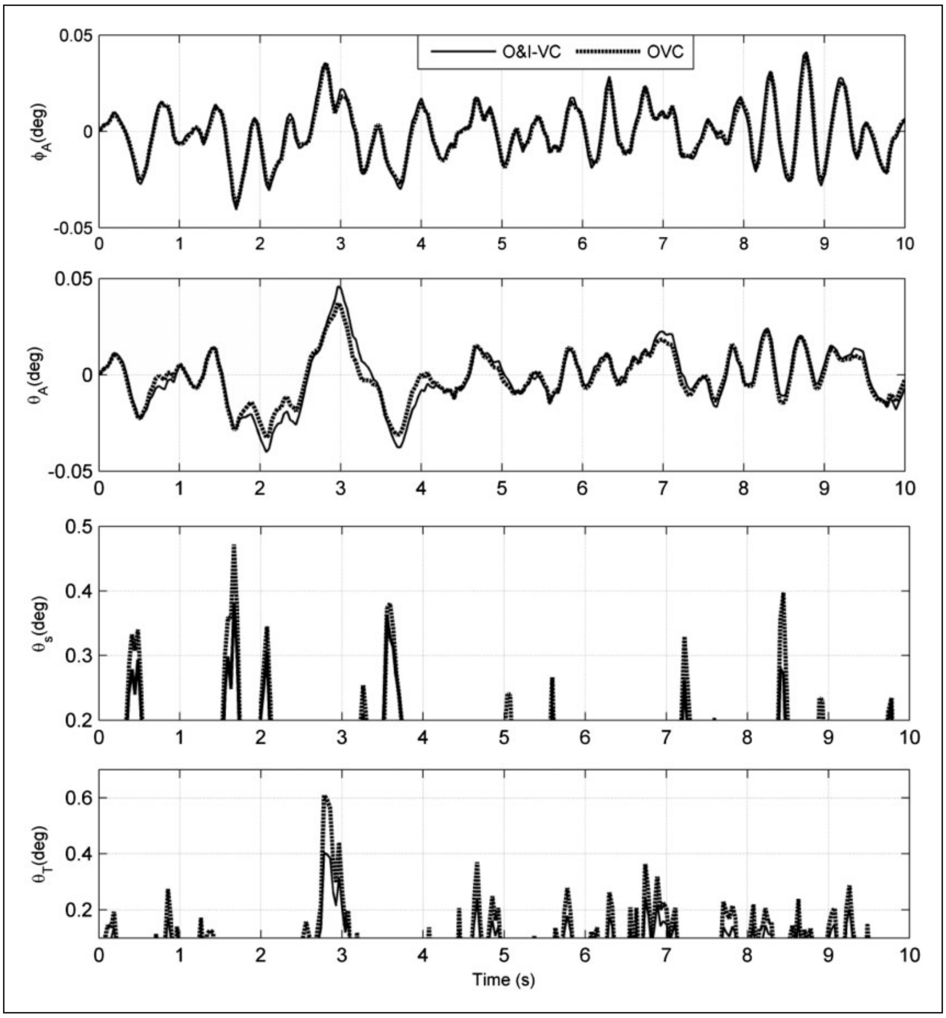

Figure 3. Closed loop responses of some of the outputs and inputs of interest (O&I-VC vs. OVC).

closed loop system. It can be easily seen from this figure that outputs out of interest do not experience catastrophic (fast and large variations) behavior. This good behavior can be explained by the exponentially stabilizing effect of O&I-VC technique.

In Figure 6, closed loop responses of one of the outputs of interest (i.e., helicopter roll angle), one of the inputs of interest (i.e., main rotor longitudinal cyclic blade pitch angle), and blade collective flapping angle for first and fourth closed loop systems are illustrated. Using unaware O&I-VC, the peak values of outputs of interest (e.g., ϕ_A) increase, while the peaks of inputs of interest (e.g., θ_s) decrease. At both situations, the other outputs (e.g., β_0) do not experience catastrophic behavior.

Conclusions

An algorithm combining two different variance-constrained control strategies named as OVC controller and IVC controller is developed. Closed loop performance of the resulting novel control technique called as combined O&I-VC controller is evaluated on air vehicles (i.e., helicopters). Linearized state-space models of them are applied in order to see feasibility and effectiveness of this new strategy (i.e., O&I-VC). Closed loop responses of some of the outputs of interest (i.e., helicopter roll and pitch angles), some of the inputs of interest (i.e., main rotor cyclic blade pitch angles and collective tail rotor angle), and some of the outputs out of interest (i.e., collective blade flapping and lead-lagging angles, longitudinal and vertical helicopter linear velocities, and lateral and vertical helicopter angular velocities) are investigated while using O&I-VC for air vehicle FCS design. After that, closed loop performance of O&I-VC is compared w.r.t. the two other classical control techniques (i.e., OVC and IVC).

Using O&I-VC technique, output and input constraints are satisfied. It is found that O&I-VC is more advantageous than OVC since it reduces peak values of control inputs while fixing peak values of outputs of interest. This means that same output performance can be satisfied with less control energy. Moreover, it is also determined that O&I-VC is more advantageous than IVC since it reduces peak values of outputs of interest while fixing peak values of helicopter control inputs. This means that same input performance can

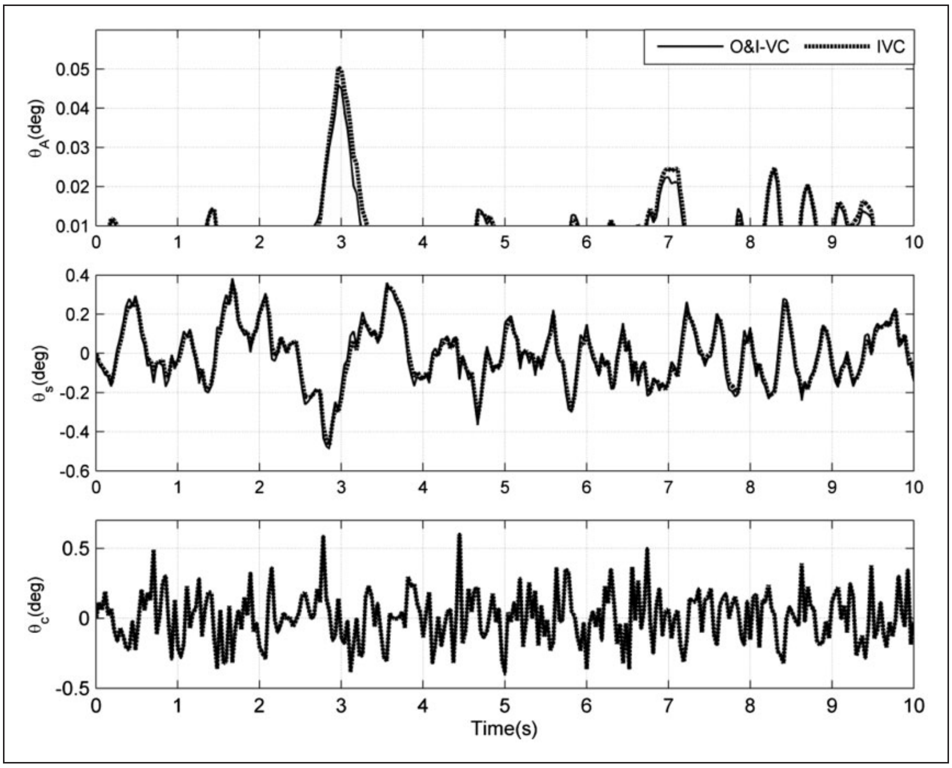

Figure 4. Closed loop responses of some of the outputs and inputs of interest (O&I-VC vs. IVC).

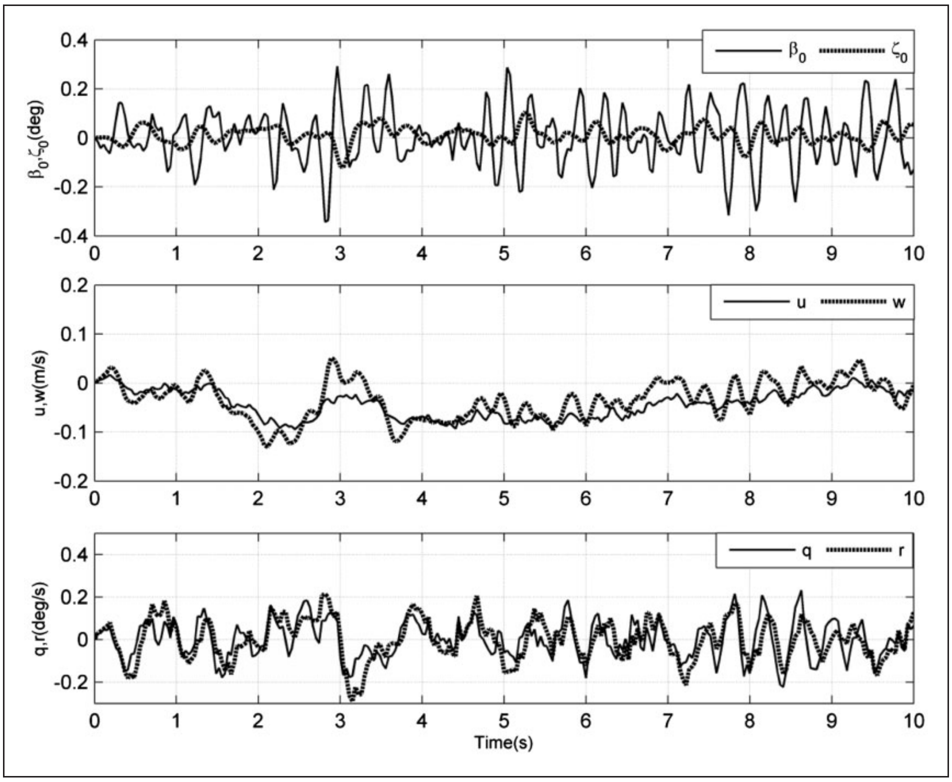

Figure 5. Closed loop responses of some of the other outputs using O&I-VC.

be satisfied with less peak values of outputs of interest.

The other outputs that are not considered for O&I-VC design do not experience catastrophic (i.e., fast and large) behavior. Finally, O&I-VC has stability robustness w.r.t. the modeling uncertainities. Using unaware O&I-VC, it is found that peak values of the outputs of interest (e.g., helicopter roll angle)

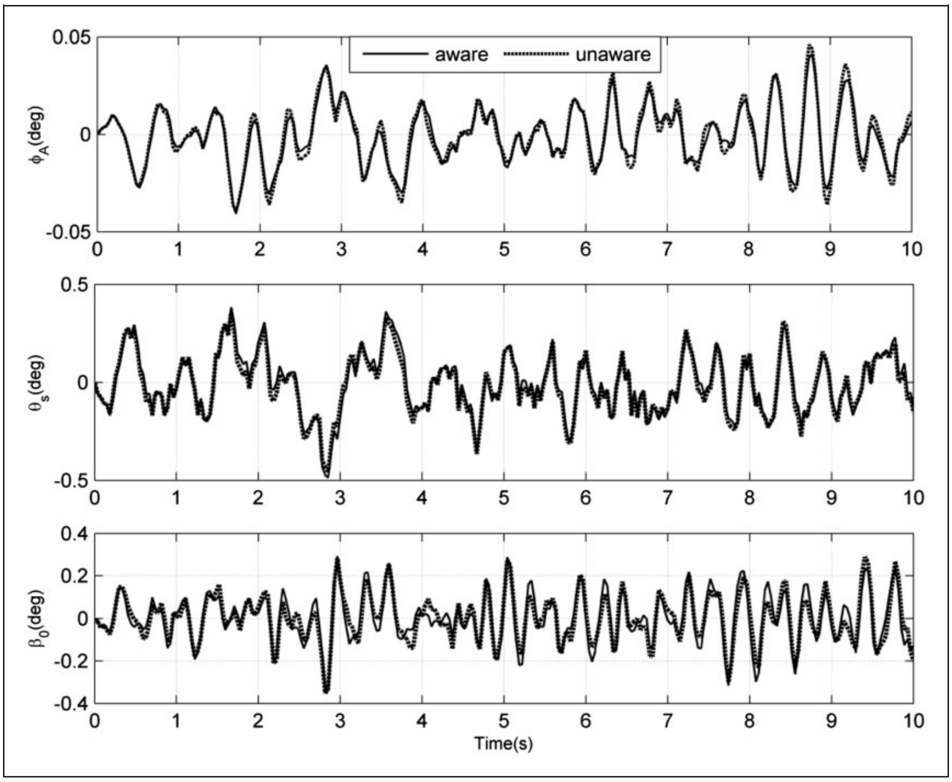

Figure 6. Closed loop responses of some the outputs and inputs using O&I-VC during uncertainty.

decrease, while the peak values of inputs of interest (e.g., main rotor lateral cyclic pitch angle) decrease. At both situations, the other outputs (e.g., collective blade flapping angle) do not experience catastrophic behavior.

Conflict of interests

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Appendix

Notation

p, q, r	helicopter angular velocities (rad/s)
u, v, w	helicopter linear velocities (m/s)
β_0	collective blade flapping angle (rad)
ζ_0	collective blade lead-lagging angle (rad)
θ_T	collective tail rotor angle, [rad]
ϕ_A, θ_A, ψ_A	helicopter Euler angles (rad)
$\theta_0, \theta_c, \theta_s$	main rotor collective and two cyclic blade pitch angles (rad)

OVC algorithm

For the given data $(A_p, B_p, C_p, D_p, M_p, \sigma_i, W, V, R, \xi_{\text{OVC}} > 0, Q_0, n > 0)$

1. Compute $\boldsymbol{X}$ and $\boldsymbol{F}$ using

$$0 = XA_p^T + A_pX - XM_p^TV^{-1}M_pX + D_pW_pD_p^T \quad \text{and} \quad F = XM_p^TV^{-1}$$

2. Compute K_j and G_j using

$$0 = K_jA_p + A_p^TK_j - K_jB_pR^{-1}B_p^TK_j + C_p^TQ(j)C_p \quad \text{and} \quad G_j = -R^{-1}B_p^TK_j$$

3. Compute X_{c_j} using

$$0 = X_{c_j}(A_p + B_pG_j)^T + (A_p + B_pG_j)X_{c_j} + FVF^T \text{ and if } \sum_{i=1}^{n_y}\left\|\sigma_i^2 - C_p(X_{c_j} + X)C_p^T\right\| \leqslant \xi_{\text{OVC}} \text{ stop}$$

4. Update $Q(j)$ with

$$q(j+1) = \left[\frac{[C(X_{c_j} + X)C^T]}{\sigma_i^2}\right]^n q(j) \text{ and go to step 2.}$$

IVC algorithm

For the given data $(A_p, B_p, C_p, D_p, M_p, \sigma_i, W, V, R, \xi_{\text{IVC}} > 0, Q_0, n > 0)$

1. Compute $\boldsymbol{X}$ and $\boldsymbol{F}$ using

$$0 = XA_p^T + A_pX - XM_p^TV^{-1}M_pX + D_pW_pD_p^T \text{ and } F = XM_p^TV^{-1}$$

2. Compute K_j and G_j using

$$0 = K_jA_p + A_p^TK_j - K_jB_pR^{-1}B_p^TK_j + C_p^TQ(j)C_p \quad \text{and} \quad G_j = -R^{-1}B_p^TK_j$$

3. Compute X_{c_j} using

$$0 = X_{c_j}(A_p + B_pG_j)^T + (A_p + B_pG_j)X_{c_j} + FVF^T \text{ and if } \sum_{i=1}^{n_u}\left\|\mu_i^2 - G_j(X_{c_j} + X)G_j^T\right\| \leqslant \xi_{\text{IVC}} \text{ stop}$$

4. Update $R(j)$ with

$$r(j+1) = \left[\frac{[G_j(X_{c_j} + X)G_j^T]}{\mu_i^2}\right]^n r(j) \text{ and go to step 2.}$$